Homework Assignment #2
Due: October 5, 2010

1. Recall the formal definition of a string from class: a string over alphabet Σ is a function from $\{i \in \mathbb{N} : 1 \leq i \leq \ell\}$ to Σ (where ℓ is some natural number called the length of the string).

Recall the formal definition of string concatenation from class: If s_1 and s_2 are strings over Σ of length ℓ_1 and ℓ_2, respectively, then $s = s_1 \cdot s_2$ is a string over Σ of length $\ell_1 + \ell_2$ defined as follows.

$$s(i) = \left\{ \begin{array}{ll} s_1(i) & \text{if } 1 \leq i \leq \ell_1 \\ s_2(i - \ell_1) & \text{if } \ell_1 + 1 \leq i \leq \ell_1 + \ell_2. \end{array} \right\}$$

If z is a string of length ℓ over the alphabet Σ, then the reverse of z, denoted z^R, is a string of length ℓ over Σ defined as follows.

$$z^R(i) = z(\ell - i + 1).$$

Use these definitions to give a careful proof that, for any strings x and y over Σ, $(x \cdot y)^R = y^R \cdot x^R$.

2. Draw a deterministic finite automaton that decides the language of all binary strings that do not contain 1010 as a substring. You do not have to prove your answer is correct. However, for each state of your automaton, write down a description of all strings that take the machine to that state.

Optional programming task (Do *not* hand this in.) Write a Java programme which, given a description of a deterministic finite automaton and an input string, determines whether the automaton accepts or rejects the string. Assume that the automaton description is given in the following format.

- The first line gives two integers separated by a space: n, the number of states, and m, the number of accepting states. (We will assume the states are labelled $1, 2, \ldots, n$ and that state 1 is the starting state.)
- The second line gives a string containing one copy of each character in the alphabet.
- The third line gives a list of the m accepting states, separated by spaces. (If there are no accepting states, this line will be blank.)
- The remaining lines describe the possible transitions. If $\delta(q, a) = q'$, there will be a line containing q, a and q', separated by spaces. The end of the transitions will be indicated by a line containing just 0.

For example, the automaton shown on page 34 of the textbook would be specified as follows:

```
3 1
01
2
1 0 1
1 1 2
2 0 3
2 1 2
3 0 2
3 1 2
0
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