York University CSE 2001 November 30, 2010

Homework Assignment #10
Due: December 9, 4:00 p.m.

1. In this question, we consider languages over the alphabet $\{0, 1, \#\}$. If n is a positive integer, let $B(n)$ be the binary representation of n (with no leading 0's). For example, $B(22)$ is the string 10110.

Let $L_1 = \{B(n)\#B(m) : n, m \in \mathbb{Z}^+ \text{ and } n > m\}$. For example, 11000#10110 is in L_1 because $B(24) = 11000$ and $B(22) = 10110$ and $24 > 22$. However, the strings 11000#111111 and 10110#11000 are not in L_1. In assignment #5, we saw that L_1 is not regular.

Let $L_2 = \{B(n)\#(B(m))^R : n, m \in \mathbb{Z}^+ \text{ and } n > m\}$. For example, 11000#01101 is in L_2 because $B(24) = 11000$ and $(B(22))^R = (10110)^R = 01101$ and $24 > 22$. However, the strings 11000#111111 and 10110#00011 are not in L_2.

(a) Is L_1 context-free?

(b) Is L_2 context-free?

If you answer yes for either language, you must give a context-free grammar for that language. You do not have to give a formal proof that the grammar generates the language, but you should give, for each variable in your grammar, a precise description of the set of strings that the variable generates.

If you answer no for either language, you must give a formal proof that the language is not context-free.