

Homework Assignment #1

Due: September 28, 2010

Along with your solutions to this assignment, hand in a *separate* sheet of paper containing your name, student number and the following declaration: "I have read and understood the policy on academic honesty on the CSE2001 course web page." Sign this paper and date it. Without this declaration, your solutions will not be marked.

1. Let $f : \mathbb{N} \rightarrow \mathbb{Z}$ be defined recursively by

$$\begin{aligned} f(0) &= 0, \\ f(1) &= 1, \text{ and} \\ f(n) &= \left\{ \begin{array}{ll} -f(\frac{n}{2}) & \text{if } n \text{ is even} \\ f(n-3) & \text{if } n \text{ is odd} \end{array} \right\}, \text{ for } n \geq 2. \end{aligned}$$

- (a) Write down an explicit formula for $f(n)$. (When I say an explicit formula, I mean that it should be easy for you to answer part (c) using your formula.)
- (b) Prove that your answer to part (a) is correct for all n .
- (c) What is the value of $f(6788436246799864)$? Explain why your answer is correct.
- (d) Let $g(k) = \sum_{i=0}^{k-1} 4 \cdot 10^i$. (For example, $g(3) = 444$ and $g(7) = 4444444$.) What is the value of $f(g(10000000000000000000000000))$? Explain why your answer is correct.

Optional programming task (Do *not* hand this in.) You could write a simple, recursive Java routine to compute f using the recursive definition above. You might want to use the `BigNum` class provided by Java, since some of the numbers will not fit in ordinary Java integer variables. Try running your code to doublecheck your answers for parts (c) and (d). How long does it take to compute these values of f ?