CSE4421: Assignment 3

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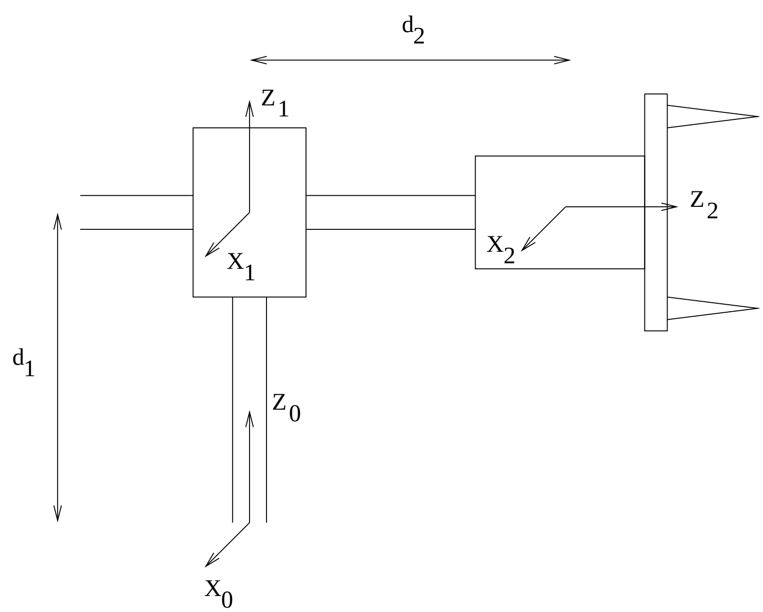
1. Consider the planar prismatic-prismatic robot shown in the following figure (joint 1 slides up and down in the page, and joint 2 slides left and right in the page).



The Denavit-Hartenberg parameters of the robot are shown in the following table.

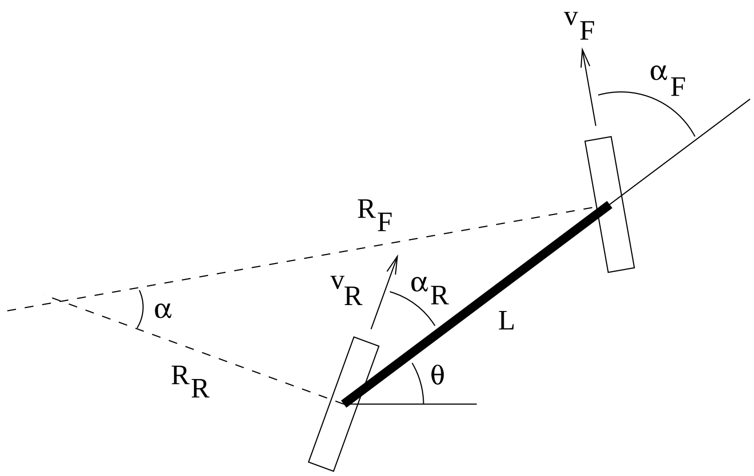
i	a_{i-1}	α_{i-1}	d_i	θ_{i-1}
1	0	0	d_1	0
2	0	$-90°$	d_2	0

(a) Compute the angular velocity ${}^2\omega_2$ of the end effector (link 2) and the linear velocity ${}^2\upsilon_2$ of the origin of frame 2; Equation 5.48 in the textbook is the appropriate equation to use. Even though the robot is planar, you should compute the velocities as 3-dimensional vectors.

(b) What is the Jacobian ${}^2J$ relating the joint velocities $\dot{d}_1$ and $\dot{d}_2$ to the linear and angular velocities?

(c) Consider only the 2×2 sub-matrix of ${}^2J$ that determines the non-zero components of the linear velocity ${}^2\upsilon_2$. Is this matrix invertible? Are there any singularities of the robot in the plane of its motion?

2. Consider a bicycle with two steerable wheels as shown in the following figure. The front and back wheels of the bicycle are connected by a frame of length L making an angle θ relative to the world X-axis. The front wheel is powered having a ground speed v_F and making an angle α_F with the bicycle frame. The rear wheel makes an angle α_R with the frame. The origin of the bicycle is located on the axle of the rear wheel.

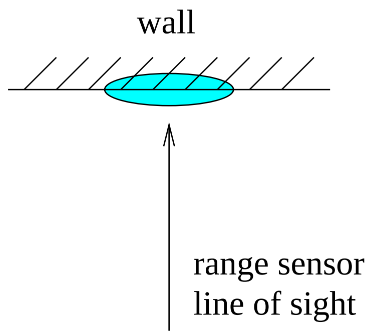


(a) Find an expression for the angle α in terms of α_F and α_R.

(b) Find expressions for R_F and R_R in terms of L, α_F, and α_R.

(c) What ground speed v_R of the rear wheel is required for smooth rolling motion?

3. Suppose you have a mobile robot with a range sensor that measures the location of a point on a solid object; its measurement noise (assumed to be additive zero-mean Gaussian) is smaller in the direction perpendicular to the object surface than in the direction tangential to the surface as shown in the figure below (the shaded ellipse is the covariance ellipse of the measurements).



The file `/cs/dept/www/course/4421/assignments/a3.mat` (also available on the Web here) contains 100 sample measurements of a single point on a wall located in some world coordinate frame; the measurements are in the world frame (not the robot frame). Copy the file into a working directory and load its contents into Matlab by typing

```
load a3.mat
```

The file contains a single variable `Q` of size 100 rows and 2 columns where each row is a measurement of the world coordinates of the point. You can compute the sample mean and sample covariance of the measurements by typing (in Matlab)

```
mu = mean(Q);
C = cov(Q);
```

(a) Estimate the coordinates of the measured point on the wall.

(b) Estimate the angle of the wall relative the the X-axis of the world frame. Show your Matlab code.

(c) Estimate the covariance matrix of the measurements in the robot frame; assume that the sensor is mounted so that it points in the positive X-direction of the robot.