Homework Assignment #16
Due: November 28, 9:30 a.m.

1. In a *mutual recurrence*, two sequences are defined together. Consider the mutual recurrence

$$
\begin{aligned}
a_0 &= 0 \\
b_0 &= 0 \\
a_1 &= 2 \\
b_1 &= 3 \\
a_n &= 2(b_{n-1} - a_{n-1} + 2n - 1) \text{ for } n \geq 2 \\
b_n &= 3(b_{n-2} - a_{n-1} + 8n - 10) \text{ for } n \geq 2.
\end{aligned}
$$

(a) Write out the first 7 values of each sequence.

(b) Guess a formula for a_n and b_n.

(c) Prove using induction that your formulas for both sequences are correct. (As in all induction proofs, clearly state the claim that you are proving by induction, the induction hypothesis, etc.)

2.

(a) Solve the recurrence

$$
\begin{aligned}
a_0 &= 5 \\
a_1 &= 1 \\
a_2 &= 41 \\
a_n &= 5a_{n-1} - 3a_{n-2} - 9a_{n-3}, \text{ for } n \geq 3.
\end{aligned}
$$

(b) Solve the recurrence

$$
\begin{aligned}
a_0 &= 2 \\
a_1 &= 7 \\
a_2 &= 12 \\
a_n &= 5a_{n-1} - 3a_{n-2} - 9a_{n-3} + 8n - 12 \text{ for } n \geq 3.
\end{aligned}
$$