Homework Assignment #5
Due: April 3, 2:30 p.m.

The front page of your solution set should be a cover page that includes only the following: your name, your student number, a list of students with whom you have discussed the problems, and a signed declaration stating "I have read and understood the policy on academic honesty on the course web page". Without this declaration, your solutions will not be marked.

1. Our goal in this question is to solve the recurrence

$$\begin{aligned} a_1 &= 0 \\ a_n &= \left(1 - \frac{1}{n}\right) a_{n-1} + 2n, \text{ for } n \geq 2. \end{aligned}$$

(a) Let $b_n = na_n$. Show that $b_n = b_{n-1} + 2n^2$ for $n \geq 2$.

(b) Solve the recurrence

$$\begin{aligned} b_1 &= 0 \\ b_n &= b_{n-1} + 2n^2, \text{ for } n \geq 2. \end{aligned}$$

(c) Give a formula for a_n.

2. Solve the following recurrence:

$$\begin{aligned} a_0 &= 14 \\ a_1 &= 31 \\ a_2 &= 28 \\ a_n &= 2a_{n-1} + 4a_{n-2} - 8a_{n-3} + 9n \text{ for } n \geq 3. \end{aligned}$$

3. For each of the following recurrence relations, give a big-O estimate of f using the Master Theorem. (Assume that f is an increasing function.)

(a) $f(n) = 2f(n/3) + 5n$ (when n is a multiple of 3).

(b) $f(n) = 4f(n/2) + 8n^2$ (when n is even).

4. Recall that $\mathbb{Z}$ denotes the set of all integers. Let $Q = \{(a, b) : a, b \in \mathbb{Z} \text{ and } b \neq 0\}$. Let R be the relation on Q where $((a, b), (c, d)) \in R$ if and only if $ad = bc$.

(a) List three elements of Q that are related to $(1, 2)$ by R.

(b) Show that R is reflexive.

(c) Show that R is symmetric.

(d) Show that R is transitive.

5. (Bonus question) Solve the following recurrence:

$$\begin{aligned} a_0 &= 0 \\ a_1 &= 0 \\ a_2 &= -12 \\ a_3 &= 28 \\ a_n &= a_{n-2} - 2a_{n-3} - 2a_{n-4}. \end{aligned}$$