

Homework Assignment #3

Due: March 3, 2:30 p.m.

The front page of your solution set should be a cover page that includes only the following: your name, your student number, a list of students with whom you have discussed the problems, and a signed declaration stating “I have read and understood the policy on academic honesty on the course web page”. Without this declaration, your solutions will not be marked.

1. Find a formula for $\sum_{i=1}^n \sum_{j=i}^n (n - i + j)$. (Show why your answer is correct.)
2. For each of the following functions $f(n)$, give a **simple** function $g(n)$ such that $f(n)$ is $\Theta(g(n))$. Prove your answer is correct.
 - (a) $f(n) = n + \sqrt{n} - \frac{\sqrt{n}}{n}$.
 - (b) $f(n) = 13n + \sqrt{n} \cdot (2 \log n)^2$.
 - (c) $f(n) = 2^{2n+5} - n \cdot 3^n$.
3. Let $f(n) = \sum_{i=1}^n i \log_2 i$. Show that $f(n)$ is $\Theta(n^2 \log n)$.
4. Read about the \log^* function on page 273 of the textbook.
 - (a) The number of electrons in the universe is believed to be less than 10^{90} . Compute the value of $\log^*(10^{90})$. Show your answer is correct.
 - (b) Prove that $\log^* n$ is not $O(1)$.
5. Consider the following recursive definition of a sequence.

$$\begin{aligned} a_1 &= 1 \\ a_2 &= 3 \\ a_n &= a_{n-1} + a_{n-2} \text{ for } n > 2 \end{aligned}$$

- (a) Which elements of this sequence are even? Show your answer is correct.
- (b) Prove that no element of this sequence is divisible by 5.