

## Homework Assignment #2

### Due: February 10, 2:30 p.m.

The front page of your solution set should be a cover page that includes only the following: your name, your student number, a list of students with whom you have discussed the problems, and a signed declaration stating “I have read and understood the policy on academic honesty on the course web page”. Without this declaration, your solutions will not be marked.

1. Suppose  $A_1, A_2, \dots, A_n$  and  $B$  are sets, where  $n \geq 1$ . Prove that  $B \cap \left(\bigcup_{i=1}^n A_i\right) = \bigcup_{i=1}^n (B \cap A_i)$ .
2. Suppose  $|A| = 100$ ,  $|B| = 500$  and  $|C| = 1000$ .
  - (a) What is the largest possible value of  $|A \cup (B \cap C)|$ ? Give an example of 3 sets  $A, B, C$  that achieve this value. Briefly explain why the value cannot be any bigger.
  - (b) What is the smallest possible value of  $|A \cup (B \cap C)|$ ? Give an example of 3 sets  $A, B, C$  that achieve this value. Briefly explain why the value cannot be any smaller.
3. Let  $f : A \rightarrow B$ . Suppose  $C$  and  $D$  are subsets of  $A$ .
  - (a) Prove that  $f(C \cap D) \subseteq f(C) \cap f(D)$ .
  - (b) Give an example of a function  $f : \mathbb{N} \rightarrow \mathbb{N}$  and sets  $C, D \subseteq \mathbb{N}$  such that  $f(C \cap D) \neq f(C) \cap f(D)$ .
4. Page 143, exercise 40.
5. Find a formula for  $\sum_{i=1}^n \sum_{j=i}^n (n - i + j)$ .