

Homework Assignment #1

Due: January 25, 2:30 p.m.

The front page of your solution set should be a cover page that includes only the following: your name, your student number, a list of students with whom you have discussed the problems, and a signed declaration stating “I have read and understood the policy on academic honesty on the course web page”. Without this declaration, your solutions will not be marked.

1. Consider the following two statements:

Statement 1: For every integer x with the property _____, there is an integer y with the property _____ such that _____.

Statement 2: There is an integer x with the property _____ such that, for all integers y with the property _____, _____.

- (a) Give three simple properties that can be used to fill in the blanks of both statements. These three properties should make Statement 1 true and Statement 2 false. (The three properties should be used in the same order in both statements.)
- (b) Give three simple properties that can be used to fill in the blanks of both statements. These three properties should make Statement 1 false and Statement 2 true. (The three properties should be used in the same order in both statements.)

2. Prove that, for all integers n , $\frac{n(n+11)}{2}$ is an integer. (Hint: do not use induction.)
3. Problem 40 on page 254 of the textbook.
4. Let n be any positive integer. Let $S_n = \{i : 1 \leq i \leq n \text{ and } i \text{ is divisible by 11 but not by 7}\}$. How many elements are in S_n ? Your answer should be expressed in terms of n . Briefly explain why your answer is correct.

You may find the following notation useful in stating your answer: If p and q are positive integers, $p \operatorname{div} q$ is the quotient $\frac{p}{q}$ rounded down to the nearest integer. For example, $7 \operatorname{div} 2$ is 3 and $12 \operatorname{div} 5$ is 2.