**Definition** Notation for upper bound (**Big-O**):

$$f(n) = \mathcal{O}(g(n))$$

if *there are* a constant $c > 0$ and a "threshold" $n_0$ so that

$$\text{for all } n > n_0 : \qquad f(n) \leq cg(n)$$

**Definition** Notation for upper bound (**little-o**):

$$f(n) = \mathcal{O}(g(n))$$

if *for all* $c > 0$, *there is* a "threshold" $n_0$ so that

$$\text{for all } n > n_0 : \qquad f(n) \leq cg(n)$$

**Definition** Notation for lower bound (**Big-Omega**):

$$f(n) = \Omega(g(n))$$

if *there are* a constant $c > 0$ and a "threshold" $n_0$ so that

$$\text{for all } n > n_0 : \qquad f(n) \geq cg(n)$$

**Definition** Notation for lower bound (**little-omega**):

$$f(n) = \Omega(g(n))$$

if *for all* $c > 0$ *there is* a "threshold" $n_0$ so that

$$\text{for all } n > n_0 : \qquad f(n) \geq cg(n)$$

**Definition** Notation for exact order (**Theta**):

$$f(n) = \Theta(g(n))$$

if both

$$f(n) = \mathcal{O}(g(n)) \qquad \text{and} \qquad f(n) = \Omega(g(n))$$

i.e., there are constants $c_1, c_2 > 0$ and a "threshold" $n_0$ so that

$$\text{for all } n > n_0 : \qquad c_1 g(n) \leq f(n) \leq c_2 g(n)$$

## Proof by Induction

| | | |
|---|---|---|
| Prove | $P(n)$ for all $n \geq 0$ | $P(n)$ for all $n \geq k$ |
| **Base case** | prove $P(0)$ | prove $P(k)$ |
| **Induction step** | I.H: $P(n)$ for $n \geq 0$<br>Prove $P(n+1)$ | I.H: $P(n)$ for $n \geq k$<br>Prove $P(n+1)$ |