YORK UNIVERSITY

FACULTY OF PURE AND APPLIED SCIENCE

2005 WINTER TERM EXAMINATION

Course Number: COSC2001
Title: Introduction to Theory of Computation

Duration: 3 hours

No aids allowed.

- There should be 8 pages in the exam, including this page.
- Write all answers on the examination paper. If your answer does not fit in the space provided, you can continue your answer on the back of a page, indicating clearly that you have done so.
- You may use Church's Thesis in your answer to any question on this exam.
- Write legibly.

Name _____

(Please underline your family name.)

Student Number _____

1. _____/12
2. _____/6
3. _____/3
4. _____/3
5. _____/4
6. _____/3
7. _____/3
8. _____/3
9. _____/2
10. _____/4
11. _____/4
12. _____/3

Total: _____/50

CONT'D...

1. [12 marks] For each of the following, you must determine whether the statement is true or false. Circle the correct answer. Then briefly justify your answer.

[2] **(a)** Every subset of a regular language is regular.

TRUE FALSE

[2] **(b)** If language L is regular then $\{xy : x \in L \text{ and } y^R \in L\}$ must also be regular. (Recall that y^R is the string y written backwards.)

TRUE FALSE

[2] **(c)** If there is a pushdown automaton with only one state that accepts the language L (using acceptance by empty stack), then L must be regular.

TRUE FALSE

[2] **(d)** The language $\{M : M \text{ is a Turing machine and } 0110 \in L(M)\}$ is recursive.

TRUE FALSE

[2] **(e)** The set of recursive languages is closed under union.

TRUE FALSE

CONT'D. . .

Question 1, continued

[2] **(f)** The language $\{M : M$ is a Turing machine that takes at least 1000 steps on the input string 0110$\}$ is recursive.

TRUE FALSE

2. [6 marks] For each of the following languages, you must determine whether the language is regular, context-free, recursive or recursively enumerable (r.e.). For each language, circle the *leftmost* correct answer. (For example, if a language is both recursive and recursively enumerable, circle "recursive".)

[1] **(a)** $\{0^i 1^i 2^i 3^i : i \geq 0\}$.

REGULAR CONTEXT-FREE RECURSIVE R.E. NOT R.E.

[1] **(b)** $\{(M, w) : M$ is a Turing machine that does not accept string $w\}$.

REGULAR CONTEXT-FREE RECURSIVE R.E. NOT R.E.

[1] **(c)** The set of all binary strings whose first and last character are different from each other.

REGULAR CONTEXT-FREE RECURSIVE R.E. NOT R.E.

[1] **(d)** $\{M : M$ is a Turing machine that accepts at least three different strings$\}$

REGULAR CONTEXT-FREE RECURSIVE R.E. NOT R.E.

[1] **(e)** $\{0^i 1^j 0^i : i, j \geq 0\}$.

REGULAR CONTEXT-FREE RECURSIVE R.E. NOT R.E.

[1] **(f)** The language containing a single string s, where $s = \left\{ \begin{array}{ll} 0110 & \text{if God does not exist} \\ 1011 & \text{if God does exist.} \end{array} \right\}$

REGULAR CONTEXT-FREE RECURSIVE R.E. NOT R.E.

CONT'D...

3. [3 marks] Give a regular expression for the set of all strings over the alphabet $\{a, b, c\}$ that contain at least one a and at most one b.

4. [3 marks] Draw a deterministic finite automaton that accepts the language described by the regular expression $11 + (10 + 00)1^*0$. Use as few states as possible.

CONT'D. . .

5. [4 marks] Let $L_5 = \{0^i 1^j : i - j \text{ is a multiple of } 5\}$. (Note that $0, -5, -10, ...$ are considered multiples of 5.) Is L_5 regular? Circle the correct answer and then prove your answer is correct.

YES NO

6. [3 marks] Give a brief, *informal* description of a pushdown automaton that accepts the language $\{a^i b^j : j \geq 3i\}$. Use acceptance by empty stack.

CONT'D...

7. [3 marks] The following grammar has one variable, S, and five terminals, $a, b, +, ^*, \emptyset$.
$S \to S^* \mid S + S \mid SS \mid a \mid b \mid \emptyset$.
(This grammar describes regular expressions over the alphabet $\{a, b\}$ that do not use parentheses or epsilons.)

Give an unambiguous grammar for the same language. The parse trees of your unambiguous grammar should accurately capture the usual order of operations for interpreting regular expressions.

8. [3 marks] Give a context-free grammar for the set of all binary strings that are *not* palindromes. (Recall that a palindrome is a string that is the same backwards or forwards.)

CONT'D. . .

9. [2 marks] Give a careful definition of what it means that a language L is reducible to another language L' (sometimes denoted $L \leq L'$ in lectures).

10. [4 marks] Let $L_{10} = \{(M_1, M_2) : M_1 \text{ and } M_2 \text{ are Turing machines and } L(M_1) \subseteq L(M_2)\}$. Is L_{10} recursive? Circle the correct answer and then prove your answer is correct.

YES NO

CONT'D. . .

11. [4 marks] Let $L_{11} = \{M$ is a Turing machine and $\epsilon \in L(M)\}$. Is L_{11} recursively enumerable? Circle the correct answer and then prove your answer is correct.

YES NO

12. [3 marks] State the result that you think is the most interesting one that was proved in this course. Explain (in one or two sentences) why you think it is interesting. If you did not find any result interesting, say so and explain (in one or two sentences) why you feel this way.