Homework Assignment #3
Due: December 5 at 5:00 p.m.

You may work on this assignment in groups of at most two students. Each group should hand in a single set of solutions.

The front page of your solution set should be a cover page that includes only the following: your name(s), your student number(s), your lecture section (A or B), a list of students with whom you have discussed the problems, and a signed declaration stating "I/We have read and understood the policy on academic honesty on the course web page". Without this declaration, your solutions will not be marked.

Note: In questions 2-5, you may use the Church-Turing Thesis.

1. Turing machines may be used not only to recognize languages, but also to compute functions mapping strings to strings. In this case, the model changes very slightly: instead of having states q_{accept} and q_{reject}, there is a q_{halt} state. When the machine starts out, it has an input string w written on the tape in the usual way (with the head of the machine at the left end of the tape, and blanks following the input). The Turing machine runs as usual until it enters the state q_{halt}. The string left on the tape (*i.e.* the string before the infinite sequence of blank squares) at this time is the value of the computed function for the input string w. If the Turing machine does not halt for input w, then we say the function computed is not defined for that input.

(a) For natural numbers x, let $f(x) = \left\{ \begin{array}{ll} 4x+3, & \text{if } x \geq 8 \\ \text{undefined}, & \text{if } x < 8 \end{array} \right\}$.

Draw the state transition diagram for a Turing machine M that computes f. The input and output for M should be non-empty strings over $\{0, 1\}$ representing unsigned, non-negative integers in binary. Both the input and output are allowed to have redundent leading 0's.

(b) Give a brief description of how your Turing machine works, in the style of Example 3.7 on page 143 of the textbook.

2. Show that the *complement* of the language $EQ_{\mathsf{CFG}} = \{\langle G, H\rangle \mid G \text{ and } H \text{ are } \mathsf{CFG}\text{s and } L(G) = L(H)\}$ is Turing-recognizable.

3. Let $L_3 = \{\langle M\rangle \mid M$ is a Turing machine and on input ε, M eventually writes a non-blank symbol on the tape$\}$. Show that L_3 is decidable.

4. Show that the class of Turing-recognizable languages is closed under concatenation.

5. Let $L_5 = \{\langle M\rangle \mid M$ is a Turing machine and for some string x, both x and $x^{\mathcal{R}}$ are in $L(M)\}$.

(a) Show that L_5 is not decidable. Do not use Rice's Theorem to answer this question.

(b) Show that $\overline{L_5}$ is not Turing-recognizable.