COSC 3101 Design and Analysis of Algorithms (Winter 2004)

M. Braverman J. Elder A. Kolokolova

Assignment 2 Cover Page

Due: Friday, February 13 at 12:00 noon

- You may choose to work in a group of two, and submit a single assignment report.
- Please read "How to Prepare Assignment Reports" from the course web page.
- Please make your answers clear and succinct.
- Please use this page as the cover page for your assignment.

Student 1:
Family Name:
Given Name:
Student #:
Email:

Student 2:
Family Name:
Given Name:
Student #:
Email:

Section to which the assignment will be returned (circle one):

- M (TR11:30-13:00)
- N (W19:00-22:00)
- P (WF16:00-17:30)

Problem	Marking Scheme	Score
1	$30 = 2 \times 15$ marks	
2	$40 = 4 \times 10$ marks	
3	$30 = 2 \times 15$ marks	
Total	100 marks	

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1. Consider the following program:

   <**precondition:** x, y, z are nonnegative integers>

   while $x \neq 0$ or $y \neq 0$ or $z \neq 0$ **do**

   if $z > 0$
   $z = z - 1$
   else if $y > 0$
   $y = y - 1$
   $z = 4$
   else
   $x = x - 1$
   $y = 4$
   $z = 4$
   endif

   endwhile

   Our goal is to prove that the program terminates.

   (a) What are the "safe" positions for the computation? Define and prove a loop invariant.

   (b) Define a measure of progress of the computation, and prove that if the precondition holds the program terminates. Hint: think of xyz as a number in base 5.

2. **CLRS 31-1 Binary gcd problem**

   On most computers, the operations of subtraction, testing the parity (odd or even) of a binary integer, and halving can be performed more quickly than computing remainders. This problem investigates the ***binary gcd algorithm***, which avoid the remainder computations used in Euclid's algorithm.

   (a) Prove that if a and b are both even, then $\gcd(a, b) = 2\gcd(a/2, b/2)$.

   (b) Prove that if a is odd and b is even, then $\gcd(a, b) = \gcd(a, b/2)$.

   (c) Prove that if a and b are both odd, then $\gcd(a, b) = \gcd((a - b)/2, b)$.

   (d) Design an efficient binary gcd algorithm for input integers a and b, where $a \geq b$, that runs in $O(\log a)$ time. Assume that each subtraction, parity test, and halving can be performed in unit time.

3. Given a list of intervals $[a_1, b_1], [a_2, b_2], \ldots, [a_n, b_n]$, where $a_1, \ldots, a_n, b_1, \ldots, b_n$ are integers. We are interested in the length of the longest interval in the union of the given intervals. For example, if the given intervals are $[0, 3], [2, 6], [7, 14]$ and $[8, 10]$, then their union is $[0, 6] \cup [7, 14]$ and the length of the longest interval in the union is $14 - 7 = 7$.

   (a) Design an algorithm to solve this problem.

   (b) Analyze its running time.

   Your algorithm must be as efficient as possible. Algorithms with running time $\Theta(n^2)$ and above will receive only partial marks.