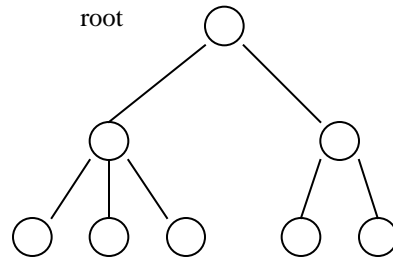


# Trees

Mar 1

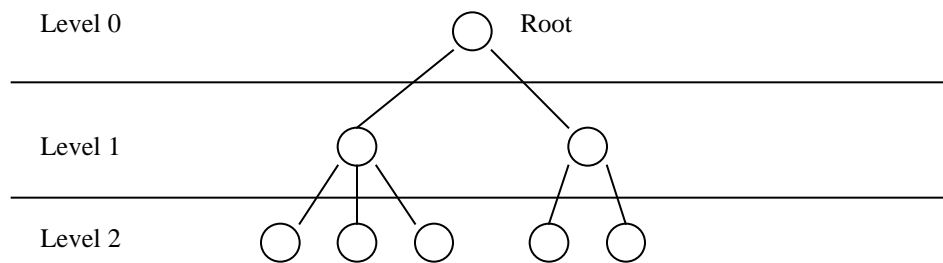


3 Sources of Terminology:

- Family Relationships (e.g. parent, child)
- Geometric relationships (left, right)
- Biological names for parts of the tree, (e.g. roots, leaves, branches)

Nomenclature:

- The **root** is the top or base of a tree structure



- A **parent** is the first node visited on path to root. It resides in the  $(i - 1)^{\text{th}}$  level.
- A **child** is the first node visited on path away from root. (can be more than one.)
- An **ancestor** is any node visited on path to root, (including root.)
- Any node that has ancestor node  $i$  is a **descendant** of  $i$ .
- All nodes that have the same parent are **siblings**

Geometric Relationships:

- A **leaf** has no children.
- An **interior** node is a node that is not a leaf.

Levels:

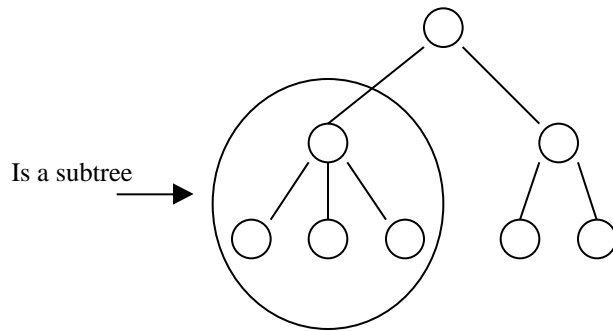
- Root is at level 0
- Children of root are at 1
- Grandchildren of root are at 2
- ...

Height:

The **height** of a tree is the number of levels it has + 1.

### Subtree:

The descendants of a node can be considered as another tree, or a subtree.



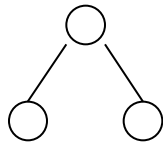
### Binary Trees:

Recursive definition:

A binary tree is either the empty tree, or a node that has left and right subtrees that are binary trees.

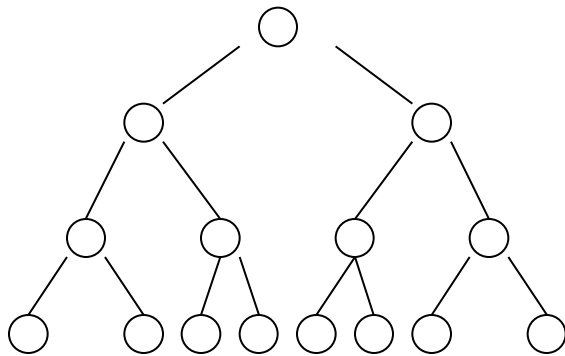
Linear definition:

A tree with no node having more than 2 children.



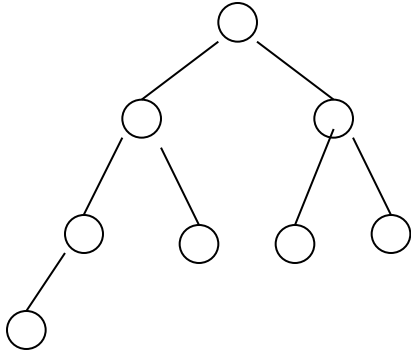
### More Definitions:

- a **full binary tree** is one in which every non leaf has two children and all leaves are at the same level.  
E.g.



- A **complete binary tree** is either full or full through the next to last level with leaves as far to the left as possible.

E.g.



This is a complete Binary Tree.

### Number of nodes and levels:

Level	# of nodes in level.	#nodes total.
0	1	1
1	2	3
2	4	7
3	8	15
4	16	31
5	32.	63

- The max height of a complete binary tree is  $O(\log n)$
- The maximum number of nodes in a tree is  $2^{n+1} - 1$ .