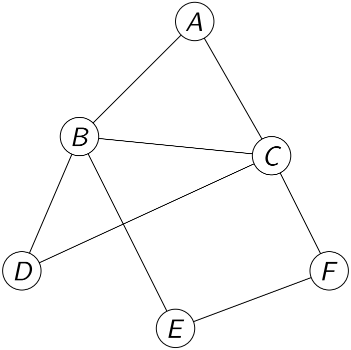
Graphs: Breadth-First Search

February 11, 2011

An Example Graph

Recall what a graph looks like.
This one is undirected and unweighted.



What is Breadth-First Search?

Breadth-first search (BFS) is an algorithm for systematically visiting all nodes of a graph that are reachable from a given "source" node s.

It also computes the minimum number of steps needed to get from s to each other node.

Applications:

- Determine if there a path from node s to node t.
- Find the portion of the graph reachable from s. (Computing connected components.)
- Find the shortest path from s to t (in an unweighted graph).
- Find a spanning tree of an undirected graph.
- And more...

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Search Algorithms: The Main Idea

Maintain two sets of nodes:

- T stores nodes that have already been visited, and
- Q stores nodes that we would like to visit in the future.

Initially, $T = \{\}$ and $Q = \{s\}$.

Repeatedly choose a node u from Q to visit next. Move u to T.

When we visit u, add u's unvisited neighbours to Q.

Pseudocode for Search

```
Q = {s}
T = {}
while Q is not empty
    remove a node u from Q
    add u to T
    for each edge u → v in the graph
        if v is not already in T, add v to Q
    end for
end while
```

If we implement Q as a FIFO queue, then this algorithm is breadth-first search.

Pseudocode for Search

$Q = \{s\}$
$T = \{\}$
while Q is not empty
    remove a node u from Q
    add u to T
    for each edge $u \longrightarrow v$ in the graph
        if v is not already in T, add v to Q
    end for
end while

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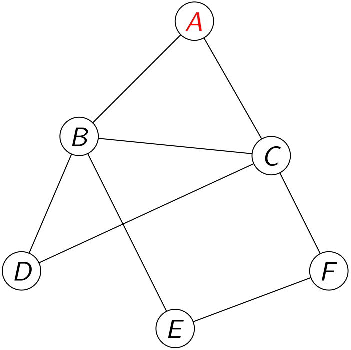
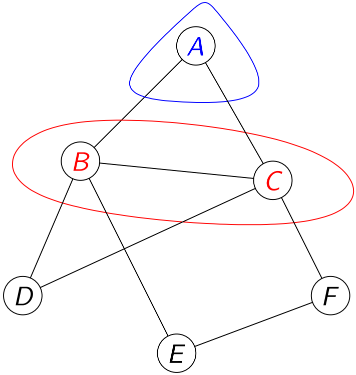
Example



$T = \{\}$

$Q = \{A\}$

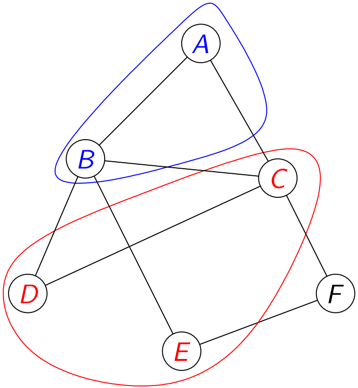
Example



$T = \{A\}$

$Q = \{B, C\}$

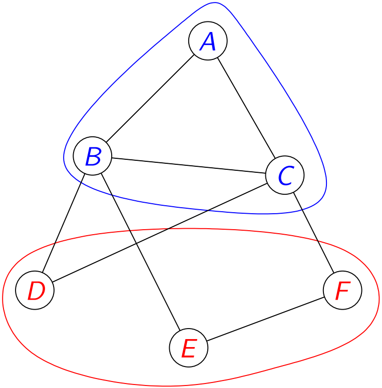
Example



$T = \{A, B\}$

$Q = \{C, D, E\}$

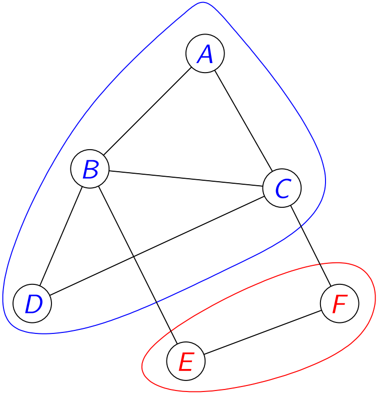
Example



$T = \{A, B, C\}$

$Q = \{D, E, F\}$

Example



$T = \{A, B, C, D\}$

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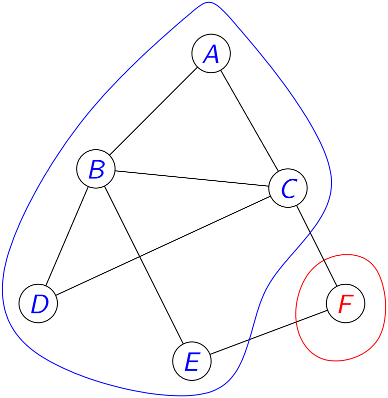
Example



$T = \{A, B, C, D, E\}$

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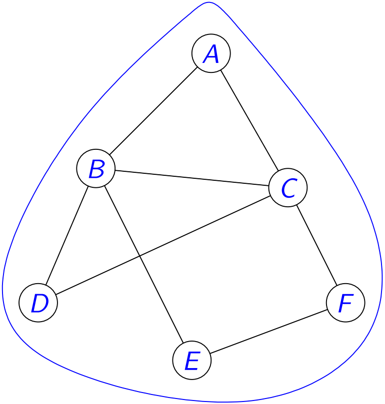
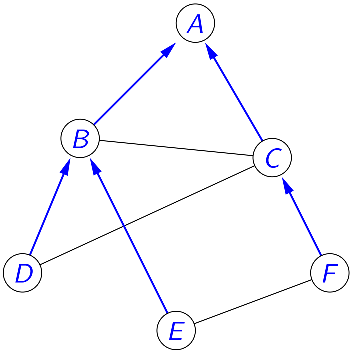


$T = \{A, B, C, D, E, F\}$

$Q = \{\}$

Shortest Path Tree



When a node v is added to Q because of edge $u \longrightarrow v$, v stores pointer to u.

This forms a tree of shortest paths towards the source of the BFS.

This also makes it easy to compute distances from source to every other node.

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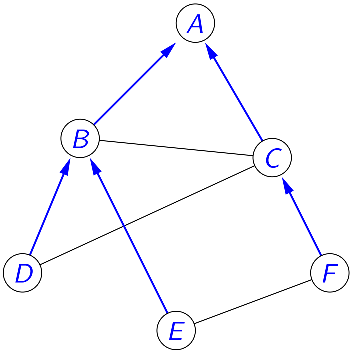

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Java implementation

`list`[i] is adjacency list of node i.
`distance`[i] is computed distance from s to i (initially -1).
`parent`[i] is parent of i in shortest path tree (initially -1).

```
Queue<Integer> Q = new LinkedList<Integer>();
distance[s]=0;
Q.add(s);
while (!Q.isEmpty()) {
    int u = Q.remove();
    for (int v : list[u]) { // for each edge u -> v
        if (distance[v] == -1) { // v not in Q or T
            distance[v] = distance[u] + 1;
            Q.add(v);
            parent[v] = u;
        }
    }
}
```

Remarks on implementation

- See last week's slides for how adjacency lists are created.
- Note that T is not represented explicitly.
- The `add` and `remove` operations of the `LinkedList` class implement a FIFO queue.
- Exactly the same code works for directed and undirected graphs.

Printing Shortest Paths

BFS can be used for computing the shortest paths from s to other nodes.

Two ways to print the shortest path from s to t (if it exists):

- Follow parent pointers from t to s and then reverse the path computed.
- Use recursion:

  printPathTo(t)
      if $t \neq s$ then printPathTo(parent(t))
      print t