Depth First Search

Suprakash Datta

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Graph algorithms - continued

Last week: All-pairs shortest paths.

This week: Depth-first search

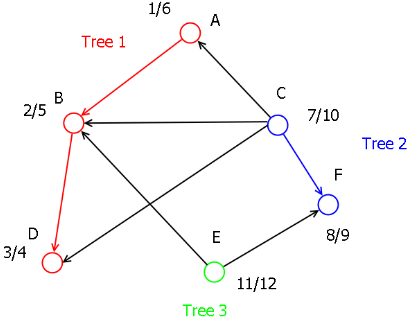
- Similar to breadth-first search
- Also used to explore graphs
- Can often use either; sometimes one is more useful than the other
- Same running time $\Theta(V + E)$, very similar implementation

The Depth-First Search algorithm

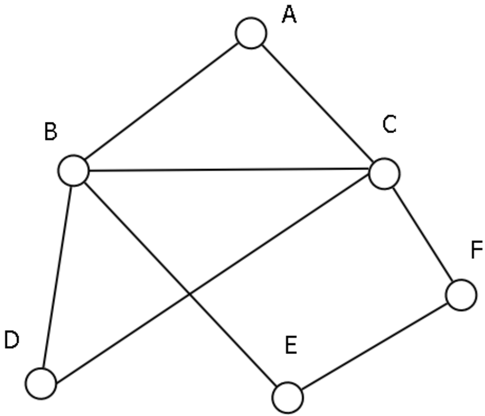
1. Start from some node s.
2. Instead of exploring all children on s before moving on (as in BFS), explore a unexplored child t of s, then to an unexplored child w of t and so on.
3. When no unexplored children exist, backtrack.
4. Initially all nodes are black, and time is zero. Thereafter time increments by 1 at each step.
5. Each node gets 2 timestamps one for when they enter the queue (turn red) and one for when they leave it (turn blue).
6. Edges can be classified by DFS – e.g. back edges go from red nodes to red nodes.
7. The above steps create a DFS tree.

DFS notes

1. Only back edges today – back edges go from red nodes to red nodes, and indicate presence of a cycle
2. Will not use start times today
3. Might need mutiple DFS trees to completely a directed graph.



An example graph



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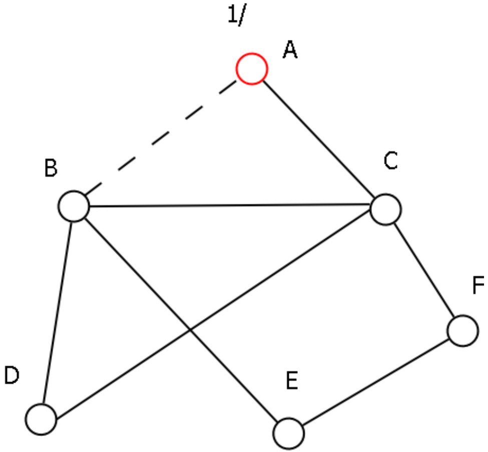

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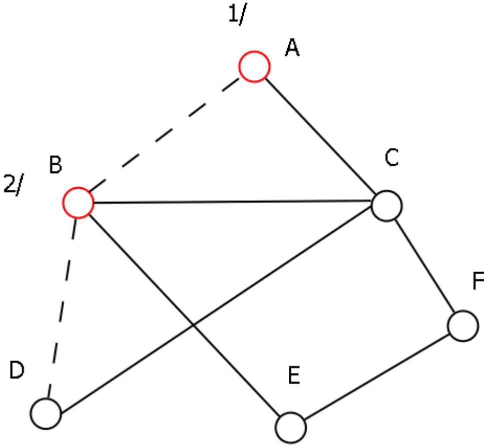

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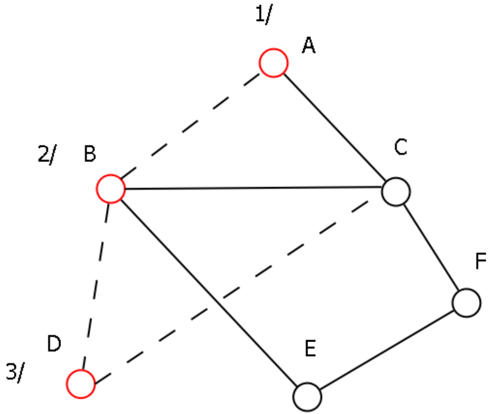

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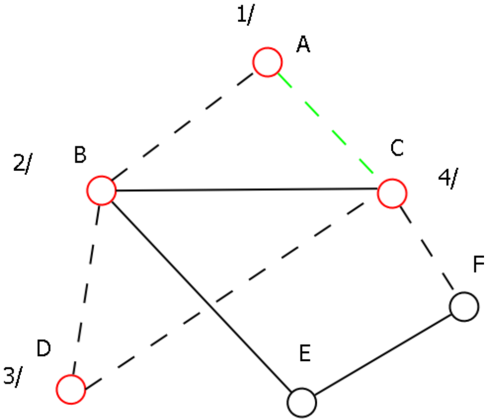
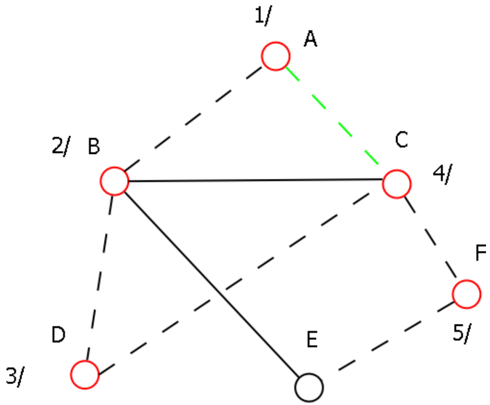


Back edge found (graph is cyclic)

An example graph



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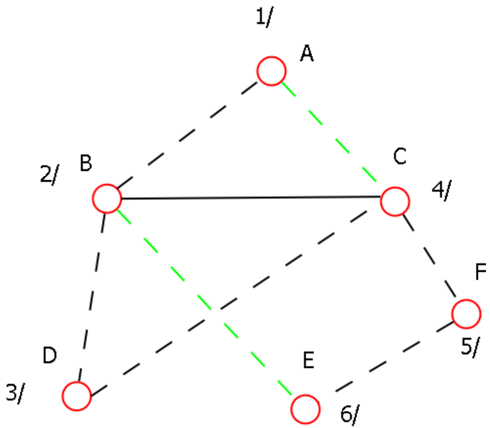

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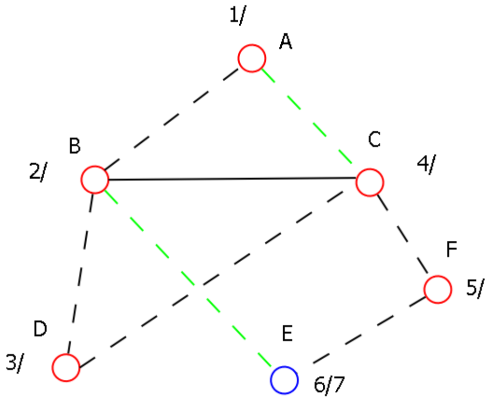

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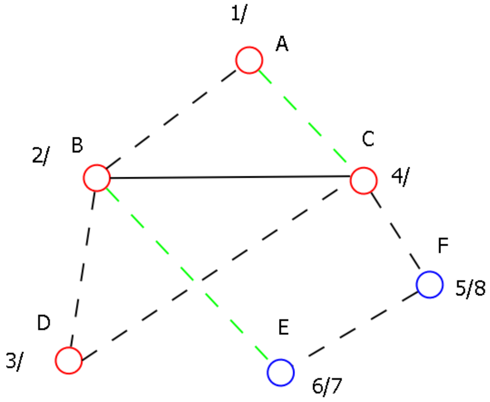

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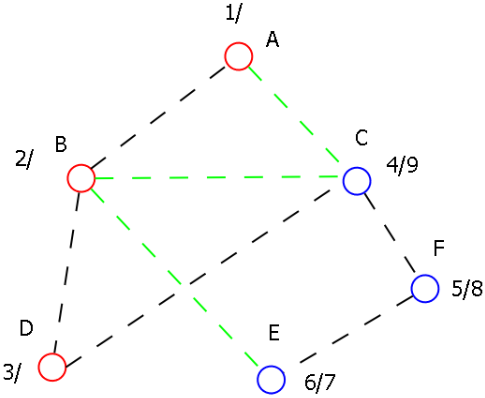

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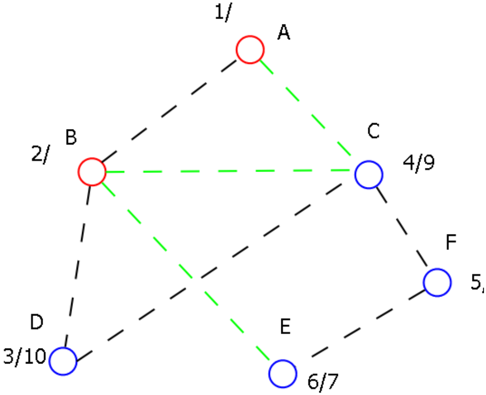

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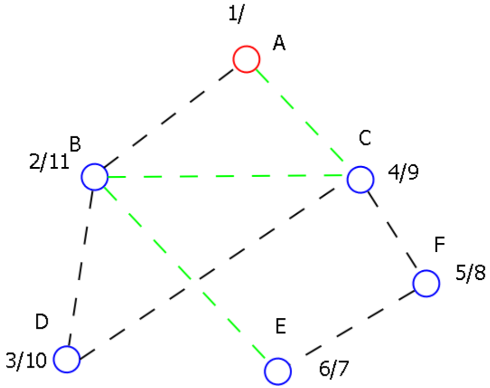

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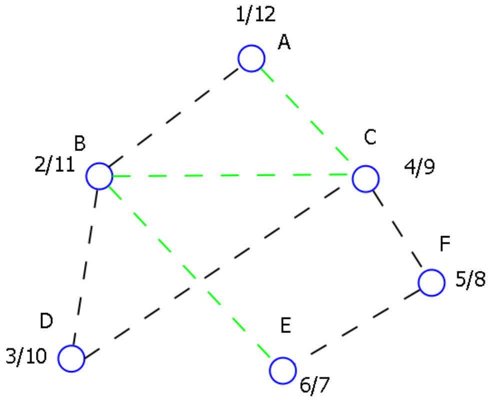
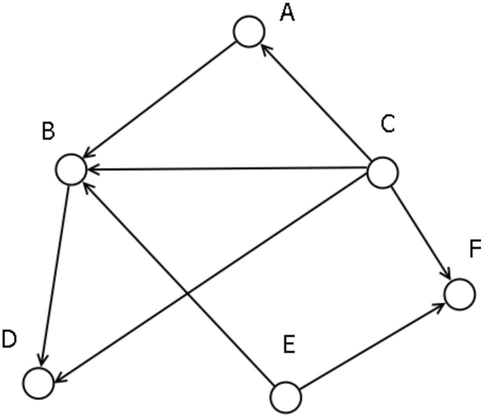


Java implementation

```
//Do a DFS starting from node s, 1 <= s <= n.
//list[i] - adjacency list of node i
//visited[], finish[]: color, finish times of nodes
Stack<Integer> stack = new Stack<Integer>();
boolean[] visited = new boolean[n + 1];
stack.push(s); // make s red
while (!stack.isEmpty())
{
   int u = stack.pop(); // make u blue
   finish[u] = ++time;
   if (!visited[u]) // u is black
   {
       visited[u] = true;
       // make black neighbours of u red
       stack.addAll(list[u]);
   }
}
```

Directed Acyclic Graphs

Directed graph with no directed cycles



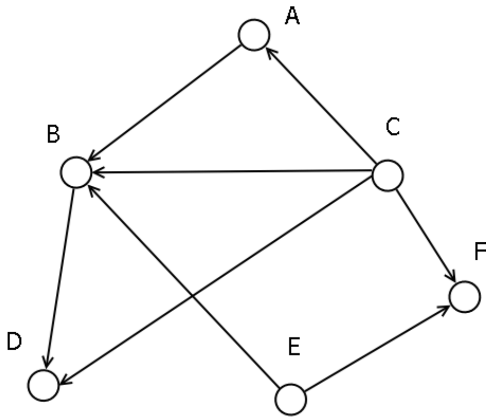
Topological Sort

Fact: A directed acyclic graph can be sorted topologically – i.e. nodes can be numbered so that all edges go from lower numbered nodes to higher number nodes.

Fact: Nodes sorted by decreasing finish times of DFS are in topologically sorted order

Fact: DFS detects cycles (presence of back edges).

An example graph



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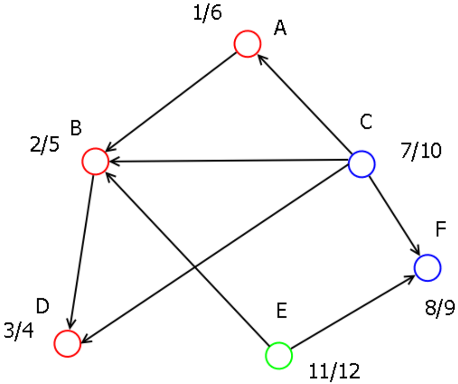

Topologically sorted order

E - C - F - A - B - D