Lecture 2. Where there is life, there are numbers: prehistory of computing

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Our present day reality is often referred to as the information age; we often speak about ourselves as consumers of digital information, as citizens of global village interconnected with computers and networks.

However, we don't need to look very hard to see that our civilization is based on more elementary concepts than computers and networks, concepts such as numbers and counting, concepts that have made our digital reality possible.

Numbers are everywhere, they surround us. We see numbers, hear numbers, read them and write them many times a day. We qualify objects using numbers: we measure them, we weight them. We speak about progress and the luck of it with numbers. We build, buy and sale with numbers. Each individual carries an assigned set of unique numbers: social security number, student number, bank account number, passport number, driver's license number, your age and weight, your height and size of shoes.

Without numbers there would be no civilization as we know it, there would be no mathematics, no computers, no social networking. It was the invention of numbers and counting that kick-started our civilization.

If the history tape of our civilization was re-wound a few million years back to the point when humans began to build social ties, would we develop our civilization in the same way? Would we invent numbers, give them symbols, start to count and perform arithmetic operations? Would we invent calculating machines and, eventually, computers? In this course I will argue that this would exactly be the case.

Man is a rational animal because he can count [Aristotle (384 BC 322 BC)].

To count, one needs the concept of a number and a purpose.

There are many important questions about our journey towards numbers and counting, about how they became first useful, and, then, indispensable and inseparable from our every-day activities, questions such as:

- when did we abstract the concept of number from quantity? when did we achieve the developmental state in which we began to comprehend that a pair of children, a pair of cows, and a pair of trees represent the same quantity – in this case a pair?
- when did we start to express these numbers using words and signs, when did we learn how to record them?
- what were the earliest of tasks that required primitive man to use numbers and counting?
- did number words emerge as the result of the need to count (as in first apple, second apple, third apple, etc) or to express quantities (as in one apple, two apples, three apples, etc.)?

Numbers are abstract concepts representing quantities. They are abstract since they do not exist in nature in the same way do cats or stones. Numbers exist only in our minds, when we read or write them, when we discuss or think about them. While we can write and read their symbols (actually created by us), we can neither see them nor touch them.

We invented numbers as abstract measures of quantities and as ordinals for counting, when there was a need to mentally operate with quantities without instantiating them, when there was a need to separate the concept of 'twoness' from 'two eggs' or from 'two trees', when counting apples turned out to be the same as counting people in a village. An aggregate of two eggs and of two trees share a quantity: a pair. Similarly, second apple arranged on the table and second person standing in a line share the same place in the order of counted objects – they immediately follow the first one. Number words allow to count men, cars, and other things (whether they exist or not) in the same way. (For more discussion on this subject see [7] and [8]).

Let us imagine early human hunters, say 50,000 years ago, making preparations for a hunt. Assuming that they didn't care too much about what kind of animals they wanted to trap, antelopes or wild pigs, how did they agree on the number of killed animals that would make their hunt a success?

They had to agree on whether a **single** large animal would do, or a smaller animal and some other animal–**a pair**–would be enough. Or perhaps, they had to agree on capturing **many** animals, where 'many' was a distinct quantity from 'single' (or one) and from a 'pair' (or two).

Since the kind of animals they were planning to hunt was not an issue, what the hunters had to agree upon was the quantity: one, two, or many. To be able to do that, the hunters had to be able to abstract (or to separate) the concept of quantity (e.g., 'one', 'two', or 'many') from particular instances of quantities (e.g. 'one pig' or 'many chicken'), that is they had to be able to envision numbers.

The process of abstracting numbers from quantities probably occurred very early in human development and over a long period of time. How early? Nobody knows, because writing was developed so late in the evolution of human culture. For instance, some of the earliest number words known in the Near East are those written by the Sumerians on clay tablets dating back to the 3rd millennium B.C. (see [7], p. 187 and Fig. 0). According to [7], "Although some Sumerian number words remain unknown, there is no possible doubt that Sumerians of the third millennium B.C. counted abstractly just the way we do."

Fig. 0. An early Summerian clay tablet. Source: The British Museum, item 140855;
http://www.britishmuseum.org/research/collection_online/collection_object_details/collection_image_gallery.aspx?partid=1&assetid=121754&objectid=327218

Abstract (symbolic) representations of concrete quantities is known to appear as early as the eight millennium B.C. Archaeological finds from Near East include little clay and stone objects called tokens. These tokens have been interpreted as accounting tools to represent quantities of animals such as sheep and cows and produced goods: bread, oil, beer, wool, and so on (see [7] and Fig. 0B).

One may also argue that the process of abstracting numbers and counting started to form at the same time as the first spoken languages as ethnographers failed to discover a single language (ancient or modern) in which the suggestion of numbers does not appear, which did not provide some way to describe or differentiate quantities. Some primitive languages provide a very limited set of numbers: just 1 and 2, or 1, 2, and many. In their 2004 paper "Exact and Approximate Arithmetic in an Amazonian Indigene Group" (see [6]), the authors detailed the numerical cognition by the Amazonian tribe Mundurukú. The Mundurukú language has only number words for 1 through 5. For other quantities the Mundurukú use the words 'some', 'small quantity', and 'many'. Although there are number words for 3, 4, and 5, they are not used consistently. For instance, the word 'five' is used for 5, 6 , 7, or even 9 objects. Only the number words for 1 and 2 are used consistently and precisely.

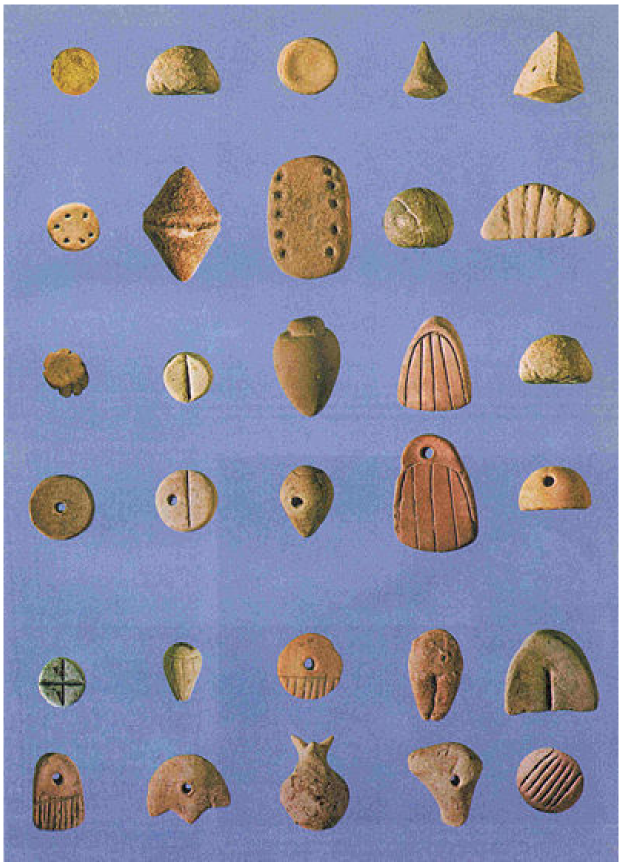

Fig. 0B. Near East tokens. Image source: unknown.

Does counting require higher intelligence?

Since we don't see advanced technology-based civilizations of insects or horses, does this mean that we are the only species on this planet that can operate with numbers and count? If we are not, what are the tasks that some animals perform guided by differences in quantities or operating with unnamed numbers?

Until a century ago or so, it was thought that man alone could count. But observations and research initiated in the first half of the last century suggests another conclusion, a possibility that species other than ourselves may use, or may be trained to use, some forms of counting.

Example 1: Birds in general
It can be observed that if a bird's nest contained say, four eggs, then one could safely be taken. But if two were removed, the bird generally abandoned the nest. This experiment does not imply that birds can count. Birds could simply distinguish between full and half-empty nest. Or, perhaps, a bird simply observes that something is wrong with the nest, perhaps that it isn't its own, and flies away.

However, another documented event may suggest that birds not only can distinguish different quantities but that they can also act upon their quantity-based observations.

Example 2: Jack vs Fred (see [1])
It was reported by Sir John Lubbock (1834-1913, British banker who made significant contribution to archaeology, botany, biology, evolutionary theory, and other scientific disciplines) that a certain man, let's call him Jack, wanted to shoot a certain crow, let's call it Fred.

To lure Fred, Jack placed a bait near a watch house. But Fred was wise and kept a fair distance from Jack and his house. So, Jack devised a clever plan of sending two men to the watch house: one returned and the other remained hidden in the watch house. But Fred realized that one man was still in the watch house and kept its distance. So, next day Jack send three men to the watch house, two returned and the third remained hidden. This too didn't fool the crow. In the end, five men were required to fool Fred the crow which, upon seeing four men returning from the watch house, wasted no time in approaching the bait. From this description we may conclude that Fred, and perhaps other crows as well, can count to 4.

One may further argue that Jack vs Fred example also suggests that some animals:

- can retain information about discrete quantities long enough to make some future decisions;

- can perform operations on quantities.

On the second point, when three man hid in the watch house and then two of them returned, how did Fred know that one was still left? For us, to determine the number of people left one would perform the subtraction

$$3 - 2 = 1.$$

But how did Fred do the math? How would Fred react if,

> first: **three** man hid in the watch house and **one** of them returned, then
>
> another **two** entered the watch house and **three** of them returned?

This puzzle requires a bit more complicated math

$$((3 - 1) + 2) - 3).$$

Animals cannot count for they lack numbers [Koehler].

In spite of the fact that some animals have developed certain forms of languages (using sounds, gestures, etc.), we have failed to find any forms of expressions used naturally by animals that would correspond to discrete quantities (or numbers). Paraphrasing a German zoologist O. Koehler, animals cannot count for they lack numbers (cf. [2]).

Koehler's experiments with birds (done before the World War II) seemed to prove that the birds can learn the concept of an "unnamed" number, perceive and act upon it.

Example 3: Koehler's Birds
Koehler successfully trained a number of bird species, including pigeons, parrots, and ravens to accept a signal indicating a small number, such as two, three, or five, and fly to a box marked by the same number, open it, and be rewarded with a meal.

What is remarkable about his experiment is that the signaled number and the number on the lid of a box were represented in a "random manner" using dots, blobs, and other distinct figures, arranged randomly. Furthermore, the number signaled to a bird was represented in a different way than that on a lid of a box, as shown in Figure 1.

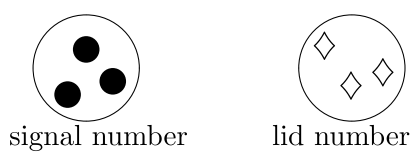

Figure 1: Sample of signal and lid number representation in Koehler's experiment.

In another experiment, Koehler trained his birds to open consecutive boxes with meal-worms until the birds collected the exact number of worms as indicated by the signal number. So, when number 5 was shown and the boxes contained the number of worms as indicated in Figure 2, then birds would stop after collecting the worm from box 5.

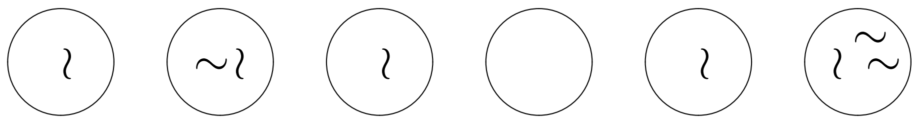

Figure 2. Distribution of meal-worms in Koehler's second experiment.

Yet another of Koehler's experiments suggests that his trained birds would pick just one worm from the 5th box even if that box contained more than 1 worm.

What Koehler's experiments indicate is that some animals can, after training, grasp the idea of an unnamed number and counting: to perform an operation an indicated number of times. Koehler stated that although training birds to "count" is possible, "we know no example of any bird counting in its normal state". (see [3] for more information on birds and counting). The behavior of solitary wasps, described below, may tell us that we simply don't know where to look for such examples.

Can Wasps Count?

Birds possess relatively large brains that allow them to learn. A certain African Grey parrot was even trained by a scientist I.M. Pepperberg to vocalize English words including numbers as well as to do some counting (see [3]). Can animals having more primitive brains, such as insects, count?

Example 3: Solitary Wasps
Solitary wasps build cells in which they lay eggs and deposit some number of caterpillars for hatched larvae to feast on. And here is the puzzle: some of these wasps deposit only one caterpillar per cell; other 10, 15, or even 24. But the number is constant in each case.

Fig. 3. Solitary potter wasp (*Eumenes fraternus*). Photograph courtesy of LARVALBUG.COM.

Fig. 4. Solitary wasp *Eumenes*, building a nest. Image from: http://threestarowl.com/art/close-in-tiny-mud-pot-forms-on-wall (no photo credit found).

There is one kind of solitary wasps (genus *Eumenes*) that does something extraordinary: if an egg in a cell is small (a male egg), the wasp deposits only 10 caterpillars; when the egg is large (a female egg), 20 caterpillars are deposited. How do all these wasps know when the job of depositing caterpillars is done for one cell and the wasp can move to stocking the next cell with caterpillars? It is not the cell size that determines that unique number of victims, as removing some of the stored caterpillars from a cell does not force the wasp to replenish the cell.

Do these wasps have some sort of fixed biological circuitry–a 'caterpillar counter'–that is reset to 0 with each new cell, which forces the wasp to move into the next cell when the counter reaches its maximum value 10 or 20? Or, perhaps, they count, they have a "build in" understanding of unnamed numbers between 0 and 24? (more on this subject in [1]).

Does counting make us unique?

All these examples seem to indicate that the ability to perceive differences between discrete quantities is not a unique feature of human intelligence, that other species are also capable of differentiating between quantities, between 1, 2, and *many*, that species other than ourselves could be taught some forms of counting. However, there are several distinct features of our intelligence that make us unique:

- **we are aware of what quantities represent**, we have abstracted the concept of numbers from quantities and progressively learned properties of these numbers;
- **we have named numbers**, invented forms of their representation including symbols to record them, to write and read such symbols;
- **we have learned to perform complex operations on numbers** and use them in diverse applications;
- **we have devised methods and designed machines to perform complex operations on numbers**, enriching our problem solving abilities.

From Numbers to Symbols

Numbers and counting became even more useful when we found ways to express numbers and, later, ways to record them.

But what came first: names for numbers as part of a language, or symbols (or gestures) to represent them? Did we first utter a sound that was to be understood, say, as 3 or did we first learn to indicate this number by showing three fingers or by placing three pebbles on the ground?

In fact there is a strong evidence that the so-called *finger method*, that is a method of indicating numbers and counting them using fingers, is a common origin of many other primitive counting methods.

The finger method seems to be as ancient as our civilization (ethnographers failed to recorded a culture where some form of the finger method would not be in use). Two fingers raised in response to questions about ”how many...” universally indicate number two, a pair.

The finger method of counting is still in use since it is simple to learn and convenient to use. Small children use their fingers to solve simple counting tasks such as counting the number of their toys or pets.

Showing fingers was not the only ancient method of non-verbal expression of numbers. There were others:

- laying sticks, pebbles, beads, or shells on the ground;
- making marks on the ground with a stick;
- cutting notches or making scratches on a stick or a bone;
- making knots on a string;
- and many, many more.

Some of these methods gave rise to number notations, others to ways of recording them.

Fig. 5. A child counting with sticks: "Little Sweets counted the Popsicle sticks and then found the number card that matched." From http://funfrugalmommy.blogspot.com/2011/04/letter-p.html.

Fig. 6. Bones with notches; do they represent primitive method of recording numbers or are they objects of ancient rituals? (no photo credit found)

Fig. 7. An Inca man holding a *quipu*, a counting and recording aid made of a complicated system of strings and cords with knots. Photograph by Crider (2011), http://peruvianpathways.wordpress.com/page/3/

For more information on the issues covered in this section, see [8].

Number Notations

When early humans first began to make their thoughts known to one another by means of speech and gestures, numbers could be named (number words) or gestured.

Laying down sticks and shells, making scratches on sticks and tying knots on cords were simple and practical ways of communicating numbers. However, the use of number words (especially for small quantities) was most convenient form of conveying numerical information as they required no external aids. And so, before the invention of writing, numbers were given names individually or in accordance with some naming convention.

Number systems came next. Individual names could be given only to a few numbers. In decimal system we have: one, two, three,..., ten, eleven, twelve,.., twenty, ..., hundred, thousand, million. But to operate with larger and larger numbers (and with very many of them) it was more convenient to create a naming convention, or a system, to name all numbers in use and to indicate how other, even larger numbers will be named when needed.

EXAMPLE 4: NAMING CONVENTION IN DECIMAL SYSTEM
In decimal system, combining ‘twenty’ and ‘one’ gives ‘twenty one’. Combining ‘thousand’ with ‘twenty one’ gives ‘one thousand and twenty one’. The rules for the creation of complex number names, such as ‘twenty one’ from individual names, such as ‘one’ and ‘twenty’, form a **number naming system**.

In decimal system, million is an individual name given to a large number consisting of 1 followed by 6 zeros. Other large numbers that are multiples of 10 may acquire individual names when needed. Looking at large numbers already named we have:

- Googol: 1 followed by 100 zeros (this number has 101 digits)
- Googolplex: 1 followed by Googol zeros (this number has Googol+1 digits)

Number systems began to appear around 3,000 B.C.–perhaps even earlier. At that time, a number of regions around the world had prosperous cities with thriving cultures, trade, and businesses, controlled by strong governments, supported by armies. The need to operate with large numbers required number naming conventions, symbolic representation of numbers, and means to record them. For these reasons, the Sumerians, Egyptians, Babylonians, Greeks, and Romans, cultures in India and China, the Mayas and the Incas developed or adopted diverse number systems and ways to record numbers.

It is not surprising that some notation systems for numbers evolved from primitive methods of expressing numbers using fingers or sticks. For instance, the number ’one’ was ”written” (stamped on a clay tablet, painted on a piece of pottery, carved in a rock) as ‘|’ or ‘–’ by many of these cultures. It is simply a representation of a lifted or pointing finger or a single stick laid down on a surface. The number ‘two’ was commonly expressed as ‘||’ (two fingers) or ”=” (two sticks, still in use in China and Japan).

Example 5: Early Chinese system of numerals
The system shown in Fig. 8 was based on the use of sticks laid upon a surface. It was used for the purpose for counting as well as in some written documents.

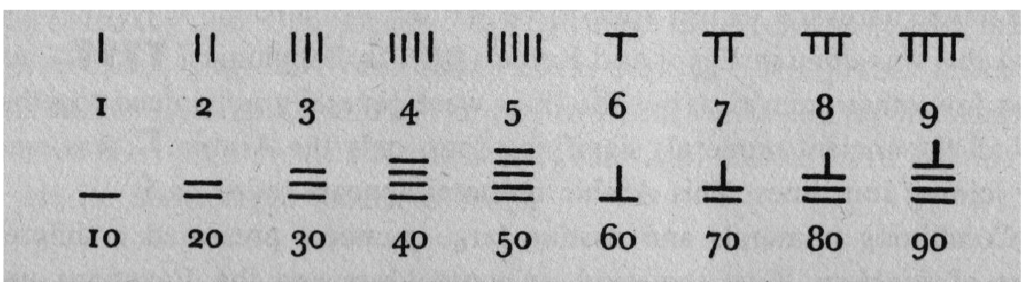

Figure 8. Early Chinese system of numerals. From [1], p. 446.

Example 6: Roman numerals.

I	**II**	**III**	**IV**	**V**	**X**	**L**	**C**	**D**	**M**
1	**2**	**3**	**4**	**5**	**10**	**50**	**100**	**500**	**1000**

So, **M M D C C C L X X X X V I I I I** written in the Roman form represents the decimal number 2,899.

Example 7: European (or Decimal) Numerals

origin: Hindu-Arabic
developed: Europe at least since 12th century

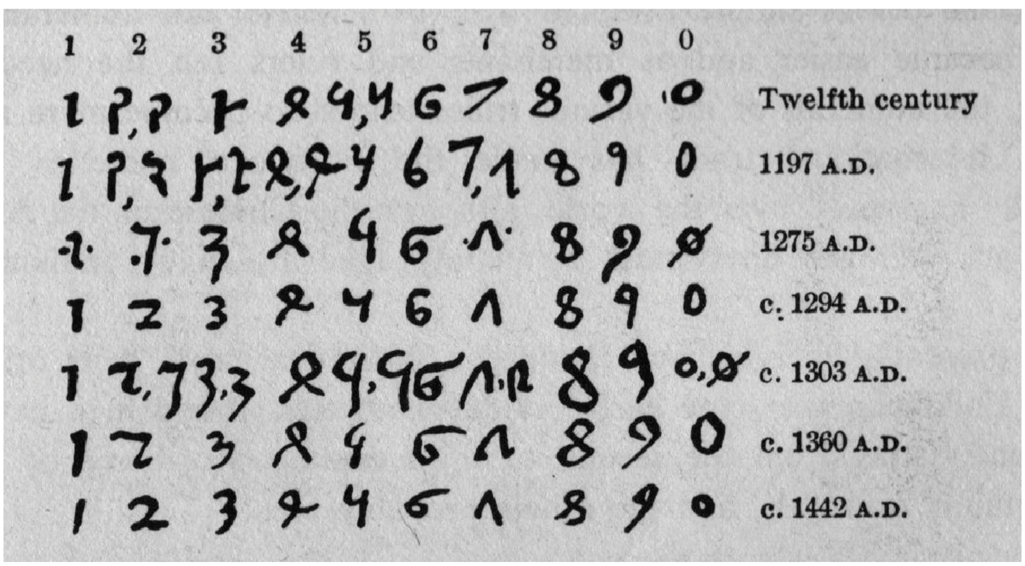

Figure 8. The evolution of the decimal notation. From [1], p. 454.

Other cultures developed symbols for "one" that resembled a pebble or a bead.

For more information on the issues covered in this section, see [8].

Operations on Numbers

The first use of numbers was simple counting:

- How many siblings do you have?
- one, two, three, four, yes – I have four siblings.

Then came the need to use numbers to express the results of human activities, such as determining the total debt to a shop keeper, and that required, apart from counting, the application of some operations on numbers, such as addition and subtraction.

Example 8: The Chicken Farmer Problem
Suppose that 3000 years ago, a certain Babylonian farmer had 5 chickens and noticed that each year his flock of chickens would double if not for the fact that he was always losing 4 either to wild animals or just to his dietary requirements (he was always keeping his loses at 4). How many chickens would he have after 3 years?

Let's see:

year 1: he starts with 5 chickens; his flock doubles to 10, minus 4 (predators and/or eating) and the total after this year is 6

year 2: he starts with 6 chickens; his flock doubles (giving 12) minus 4 (predators/eating) results in 8;

year 3: he starts with 8 chickens; his flock doubles (giving 16) minus 4 results in 12.

OK, after 3 full years of chicken farming, he will end up with 12 birds. In fact, the above process of calculation (algorithm) requires not only the knowledge of simple arithmetic operations, such as addition (to calculate the doubling of the flock) and subtraction (- 4) but also some means of storing intermediate results of the calculation: 5, 6, and 8.

The need to perform more complex arithmetic operations on numbers and to store the results efficiently provided a test for number systems. Some systems of notation allowed for space-efficient representation of numbers (decimal) and some not (Roman).

EXAMPLE 10: Representing Numbers Using Roman and Decimal Format.
The number 2,303,888 in decimal format is using just 7 digits. The same number in Roman format requires the sequence of 23 symbols representing 100,000 (say (((|)))) followed by the sequence of 15 symbols: M M M D C C C L X X X V I I I. The entire number written in Roman format takes a substantial amount of space:

(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))
(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))(((|))))
(((|)))) M M M D C C C L X X X V I I I

Furthermore, to determine what number is represented by this string of symbols requires addition.

Some systems allowed for an easy way to express and perform all basic arithmetic operations (decimal) but not all (one can easily add and subtract using Roman numerals but the multiplication and division is complicated).

Calculation Aids

An average individual can mentally perform basic arithmetic operations only on small numbers. The use of large numbers in record keeping, trade, and science, required calculation aids.

Figure 9: This picture showing two methods of calculation: pen-and-ink method (left, performed by a calculator) and a counting board (right), appeared in *Margarita Philosophica*, by Gregorius Reich, in 1503.

For efficient calculations, the Romans used special purpose counters on a board called counting boards or tables. These counters were flat surfaces with parallel grooves etched in them. Numbers: units, fives, tens, and so on were represented by beads placed in specific grooves as shown in Figure 9 on the right.

To perform calculations, such as addition and subtraction, the beads were moved along the grooves. The number shown on this counting board is 1241 and the person is most likely attempting to add 32 to this number.

Counting boards, their variants, and refinements were developed independently in many regions of the world. For instance, counting boards were known to natives in Peru and other regions of South America long before the European conquerors arrived. In a Spanish work of 1590, a Jesuit priest, Joseph de Acosta, recollects Indians of Peru performing calculations using a counting board this way:

> In order to effect a very difficult computation for which an able calculator would require pen and ink... these Indians [of Peru] made use of their kernels of wheat. They place one here, three somewhere else and eight I know not where. They move one kernel here and three there and the fact that they are able to complete their computation without making the smallest mistake. As a matter of fact, they are better at calculating what each one is due to pay or give than we should be with pen and ink.

Source: Henry Wassén, The Ancient Peruvian Abacus, Göteborg, 1931; see also [1], pp. 462–463.

Various forms of counting boards were helping us with calculations for many centuries. Another, the so-called pen-and-ink method of calculation is still being taught to children to this day. In the next lecture we shall take a look at calculators – the calculating devices that originated from counting boards and the pen-and-ink method.

Pebbles and tally marks in the digital era

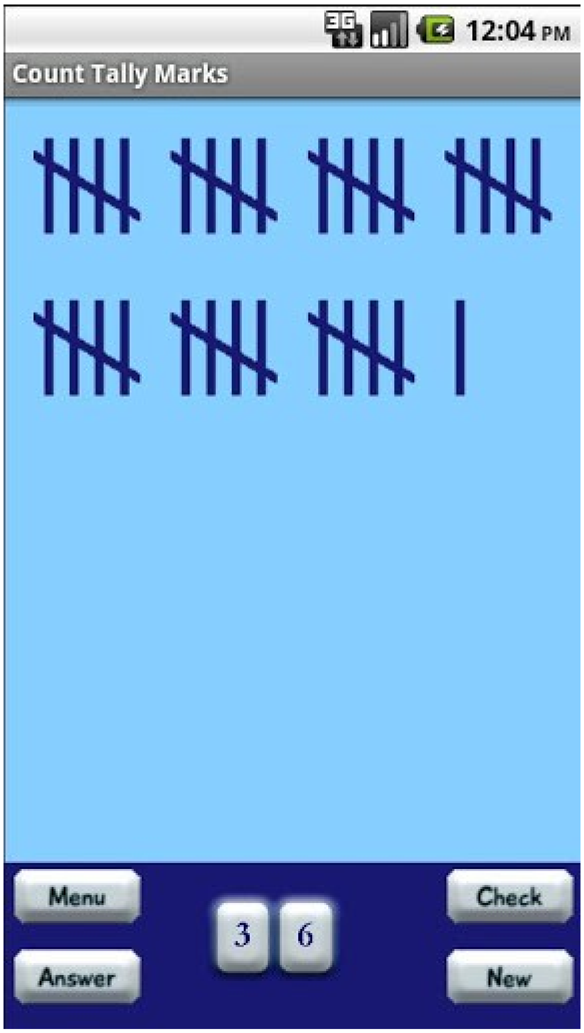

Figure 9. The JRS Tally Marks program by JRS Education.

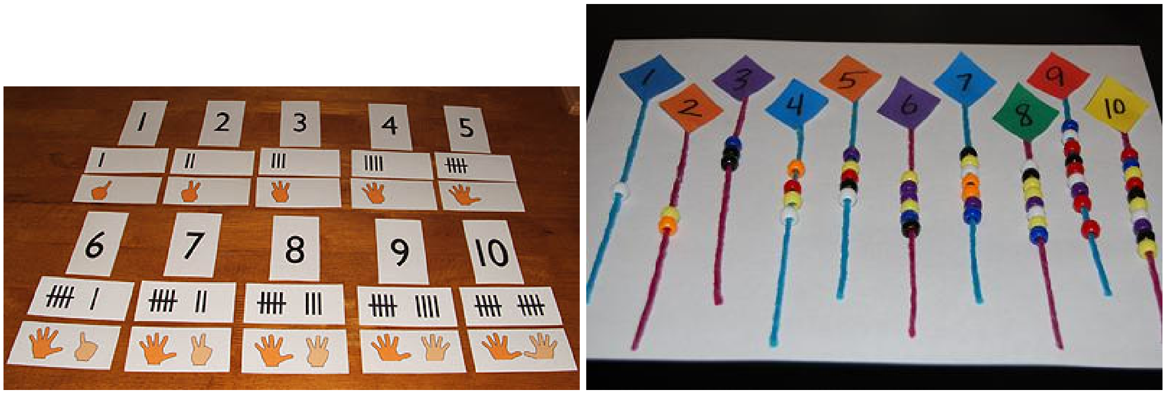

Figure 10. Counting aids, from tally marks to number symbols. Images from Spinner's End Primary (at The Linton Academy)
http://thelintonacademy.blogspot.ca/2010/05/weekly-wrap-up.html
and The Preschool Toolbox Blog
http://thepreschooltoolboxblog.com/creating-a-foundation-for-math-success-in-preschool-kindergarten

Figure 11. Tally marks in a tavern: counting pints of beer. Photograph by Phil Cook, 2012.

Figure 12. Pebbles and numbers. Image from
http://journey2excellence.blogspot.ca/2011_03_01_archive.html

Figure 13. Calculating sand volume carried by individuals using colored stone beads in Dhaka, Bangladesh, in 2007. Photographs by Ershad Ahmed. http://dhakadailyphoto.blogspot.ca/2007/02/abacus-diversion-counting-tablets.html

Additional readings

[0] L.L. Conant. *The Number Concept: Its Origin and Development*, MacMillan and Co., 1896.

[1] L.L.Conant, Counting. In *The World of Mathematics*, vol. 1, J.R. Newman (ed), 1956, pp. 432-441.

[2] O. Koehler, The Ability of Birds to "Count". In *The World of Mathematics*, vol. 1, J.R. Newman (ed), 1956, pp. 489-496.

[3] Jacky Emmerton, *Birds Judgments of Number and Quantity*, http://www.pigeon.psy.tufts.edu/avc/emmerton/

[4] *The Quipu, Pre-Inca Data Structure*, http://agutie.homestead.com/files/Quipu_B.htm

[5] Henry Wassén, *The Ancient Peruvian Abacus*, Göteborg, 1931; see also [1], pp. 462–463.

[6] P. Pica, C. Lemer, V. Izard, and S. Dehaene, Exact and Approximate Arithmetic in an Amazonian Indigene Group. *Science*, vol. 306, 15 Oct. 2004, pp. 499–503.

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