Knowledge organization and its role in representation and interpretation for time-varying data: the ALVEN system

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The so-called "first generation" expert systems were rule-based and offered a successful framework for building applications systems for certain kinds of tasks. Spatial, temporal, and causal reasoning, knowledge abstractions, and structuring are among topics of research for "second generation" expert systems. It is proposed that one of the keys for such research is knowledge organization. Knowledge organization determines control structure design, explanation and evaluation capabilities for the resultant knowledge base, and has strong influence on system performance. We are exploring a framework for expert system design that focuses on knowledge organization, for a specific class of input data, namely, continuous, time-varying data (image sequences or other signal forms). Such data are rich in temporal relationships as well as temporal changes of spatial relations, and are thus a very appropriate testbed for studies involving spatio-temporal reasoning. In particular, the representation formalism specifies the semantics of the organization of knowledge classes along the relationships of generalization/specialization, decomposition/aggregation, temporal precedence, instantiation, and expectation-activated similarity. A hypothesize-and-test control structure is driven by the class organizational principles, and includes several interacting dimensions of search (data-driven, model-driven, goal-driven temporal, and failure-driven search). The hypothesis ranking scheme is based on temporal cooperative computation, with hypothesis "fields of influence" being defined by the hypothesis' organizational relationships. This control structure has proven to be robust enough to handle a variety of interpretation tasks for continuous temporal data. A particular incarnation, the ALVEN system, for left ventricular performance assessment from X-ray image sequences, will be summarized in this paper.

Key words: knowledge representation, expert systems, medical consultation systems, time-varying interpretation, knowledge-based vision

Les systèmes experts dits de "première génération" étaient basés sur des règles et offraient un cadre intéressant pour la construction de systèmes d'application effectuant certaines tâches particulières. Le raisonnement spatial, temporel et causal, l'extraction et la structuration de la connaissance sont parmi les axes de recherche considérés pour les systèmes experts de "seconde génération". On suggère que l'une des clés de ce type de recherche soit l'organisation de la connaissance. L'organisation de la connaissance conditionne l'élaboration de la structure de contrôle, les capacités d'évaluation et d'explicitation de la base de données qui en résulte, et a une très forte incidence sur les performances du système. Nous explorons un cadre d'élaboration des systèmes experts qui s'intéresse particulièrement à l'organisation de la connaissance pour une classe spécifique de données analysées: les données continues temporalisées (séquences d'images ou autres formes de signaux). De telles données sont riches en relations temporelles de même qu'en modifications temporelles des relations spatiales et offrent ainsi un cadre d'étude approprié pour les recherches impliquant le raisonnement spatio-temporel. En particulier, la représentation facilite et renforce la sémantique dans l'organisation des catégories de savoir en fonction des rapports entre généralisation/ spécification, décomposition/agrégation, ordre temporel, instantiation, et prévision des similarités. Une structure de contrôle par hypothèses et tests est guidée par les principes d'organisation catégorielle et comprend plusieurs dimensions complémentaires de recherche (recherche guidée par les données, par modèle, par but, par échec, et temporelle). Le schéma principal d'hypothèse est basé sur une évaluation prenant en compte la temporalité, où les "champs d'influence" d'une hypothèse sont définis par ses liens organisationnels. Cette structure s'est avérée suffisamment solide pour effectuer une variété de tâches d'interprétation de données temporelles continues. Une réalisation particulière, le système ALVEN, qui évalue le fonctionnement du ventricule gauche à partir de séquences d'images radiographiées, sera présenté dans cet article.

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Mots clés: représentation de la connaissance, systèmes experts, systèmes de diagnostic médical, interprétation temporalisée, vision raisonnée.

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1.0 Introduction

A brief overview of the ALVEN application domain and the solution strategy is in order before detailed discussions are presented.

The domain of application of ALVEN is that of the evaluation of the dynamics of left ventricular tantalum marker implants from X-ray image sequences. ALVEN is thus both a visual motion understanding system as well as an example of artificial intelligence applications in medicine. The goal is to analyse both pre-operative (without markers, using contrast media) and post-operative marker films (following coronary bypass sur-

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gery), to evaluate the efficacy of surgery, locally and globally, quantitatively and qualitatively, over the recovery period (several months), and to evaluate the effects of drug interventions. It is crucial for such comparisons of perhaps subtle changes that a rich representation involving both qualitative and quantitative be obtained for each film. Other examples of computer analysis of marker implants are presented in Gerbrands *et al.* (1979), and Alderman *et al.* (1979), which addresses the problem of point of reference.

The evaluation of left ventricular (LV) performance by computer from cine representations of LV dynamics is a difficult and long-studied problem. A large number of heuristics have been proposed for measuring shape changes (Brower and Meester 1981), following anatomical landmarks (Slager *et al.*)

1981), computing segmental volume contributions (for a comparison, see Gelberg et al. (1979)), etc., all performing with varying degrees of success, but being applied independently of each other. Although such heuristics are indeed valuable quantitative measures, we propose that their limited performance is due to two key considerations: (1) it is unlikely, given the complexity of the domain of LV dynamics and the amount of training that a clinical specialist in this area receives, that any single heuristic can capture all the important facets of the evaluation and be successful in all applications; (2) the heuristics are purely quantitative in nature, contrasting with the fact that clinicians, and for that matter humans in general, deal in qualitative or descriptive terms combined with numerical quantities. That is, relational quantities are necessary components of the interpretation process, while numerical ones are secondary. The key here is that a computer system that is to solve the difficult problems present in the domain of LV dynamics interpretation must integrate the above-mentioned numerical heuristics as well as consider the symbolic processing aspects of the interpretation.

We propose that the current limited success of computer assisted analysis of left ventricular dynamics is due to three main reasons: (1) there is a strong tendency to remain within the realm of mathematical modelling for LV dynamics, and it is not at all clear that this is an adequate approach; (2) in places where mathematical models alone may be insufficient, current research into more sophisticated schemes is not yet complete, and thus, more basic research is required, particularly into representations of knowledge and interpretation control structures, before applications such as LV performance can be solved, a view also stated in Boehm and Hoehne (1981); (3) there is a distinct lack of knowledge about LV dynamics, in conjunction with disagreements about what is important to model and what terminology is to be used. We distinguish our approach from those whose goal is to provide some intermediate visual representation that must still be subjectively interpreted by a clinician (the work described in Hoehne et al. (1980) is a particularly good example of such a representation). Our goal is to perform this interpretation, in much the same way as the clinician does, and to do it in an objective and consistent

The strategy we adopted for the solution of this problem follows. In reality, there are two major problems to be solved: (1) the problem of understanding visual motion given a set of primitive image tokens over time, and (2) the problem of reasoning about spatio-temporal relationships in the context of human left ventricular dynamics.

There are three major points to be made regarding the ALVEN methodology. The first deals with the construction of the knowledge base. ALVEN's knowledge base is made up of frame-like objects called *classes*, which are organized using the relationships IS-A, PART-OF, SIMILARITY, and Temporal Precedence. These are described later in the paper. The definition of the knowledge classes requires two stages. The first is to define the general knowledge that pertains to the task of motion understanding. This must be done in such a way as to satisfy the following: (a) a set of motion classes must be found that is sufficient for use in defining the motion classes of the problem domain—problem specific motion classes are defined in terms of the general ones; and (b) the interface between image-specific concepts and general knowledge concepts is defined to be the leaves of the PART-OF hierarchy of motion concepts, and the image-specific procedures must be known.

That is, the only concepts that can be leaves of the PART-OF hierarchy are those that are directly extractable from the signal input. The second stage of knowledge class definition is to use the general purpose motion knowledge (which is a knowledge base, fully usable, in its own right), and define concepts specific to left ventricular dynamics in terms of the general ones. The problem specific knowledge base of ALVEN therefore simply 'hooks' onto the general purpose one through the knowledge organization relationships. Other motion problem domains could be handled in a similar manner.

The second aspect of ALVEN to be introduced is the control strategy. A cyclic process was defined that integrates the different search schemes into a coherent whole (see Fig. 1). Definitions of these search schemes appear later in the paper. Perhaps the most interesting aspect is how the different search schemes interact and are coordinated, and this is clear in Fig. 1. The basic cycle involves the extraction of tokens from the input signal, instantiating those tokens as leaves of the concept PART-OF hierarchy, following that hierarchy in a data-directed fashion to activate new hypotheses that are aggregates of the input tokens, thus obtaining an initial set of hypotheses. That initial set is then specialized by downward traversal of the IS-A hierarchy (one level), selecting either the most likely of the specialization, through a priori knowledge, or a random one. This intermediate set is elaborated by the activation of all component hypotheses (elements of the model specified by each class). Each hypothesis of this refined set is then matched with data or other instances. Matching successes lead to further specializations and elaborations, while failures lead to selection of alternate hypotheses via the failure-directed mechanism, which are to be considered in parallel with the failing hypotheses. Once refinements of this sort are complete (there are no more model elaborations, that is, each PART-OF subtree for each hypothesis has been fully activated) and there are no specializations (no successful matches are found for any specializations of any hypothesis), the hypotheses are ranked within competing sets by their updated goodness of fit measure, or certainty: For some hypotheses, that goodness of fit displays sufficient confidence so that the hypothesis is instantiated. Other hypotheses are deleted from further consideration. The best hypotheses are used to produce a set of predictions for the next data sample. Those predictions may be of events that are expected to occur next, or may be components of hypotheses that should have been observed but have not (differentiating between temporally-directed predictions in the former case and model-directed ones in the latter). The predictions are projected from hypothesis space to the signal domain and are used by the token extraction process as guidance. If tokens cannot be found from some prediction, a relaxation of constraints occurs, thus generalizing the prediction by moving upwards along the IS-A hierarchy from the hypothesis responsible for the faulty prediction. A 'blind' token-finding procedure is available for cases where no predictions allow for successful token finding.

The process by which time-varying matching evidence is accumulated and integrated over time is the final aspect to be introduced. This is based on relaxation labelling processes (Zucker 1978); the relaxation process described by Zucker, however, is modified in several important ways. Details of the temporal cooperative process are beyond the scope of this paper but are presented in Tsotsos (1984). The important points follow. Firstly, it should be clear that it is meaningless to talk about the certainty of a hypothesis using just its spatial evidence—space and time are considered together and cannot

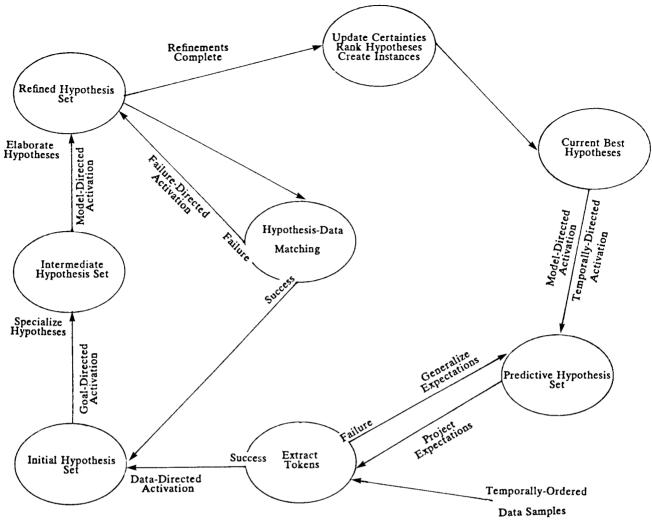


Fig. 1. The control strategy

be separated. Hypothesis certainty depends on the accumulation of spatial evidence exhibiting temporal consistency and continuity over a time interval. It also depends not only on individual hypothesis consistency but also on comparison with the evidence for other hypotheses—in other words, both local and global evaluations play a role. Relaxation methods operate over networks of local processes where processes are connected to other processes via weighted links or communication pathways. The determination of the weights is a difficult problem. In addition, the one that we use is non-linear. Relaxation schemes typically require many iterations before they converge to stable solutions (they are hill-climbing schemes). In a time-varying interpretation situation, this is not a desirable characteristic. We require that the process converge to solutions within a small fixed number of iterations and thus must discover the conditions under which relaxation schemes satisfy this requirement. This must be so because new data are being added to the interpretation as the interpretation proceeds. Our temporal cooperative process is unified with the remainder of the control scheme because the local processes are hypotheses, and the communication pathways are the knowledge organization relationships. Each relationship has an associated weight (whose value is time-varying) that is related to the semantics of that relationship. Thus, each local process (hypothesis) has a field of influence (or neighbourhood, using relaxation terminology) that is defined by its semantic relationships to other processes. A simple numerical inequality that was determined empirically must be satisfied by the weights in order for the required convergence property to be satisfied.

The remainder of the paper describes details of the representation, the control and reasoning strategies, the domain of left ventricular dynamics, and briefly presents an example of a complete analysis. Implications of the methodology conclude the discussion.

2.0 Overview of the representational scheme

2.1 Knowledge packages: classes

Packaging up knowledge leads to a modular representation, with all the advantages of modularity, particularly the enhancement of clarity and flexibility. Most knowledge-package representation schemes borrow strongly from Minsky (1975). Our frames are called *classes*. A class provides a generalized definition of the components, attributes, and relationships that must be confirmed of a particular concept under consideration in order to be able to make the deduction that the particular concept is an instance of the prototypical concept. Classes also have embedded, declarative control information, namely exceptions and similarity links. These features will be described shortly. Note that there is a distinction between the 'prerequisites' of the class, those components that must be observed in order to

instantiate the class, and the 'dependents' of a class, those components that must be derived on instantiation. Dependent slots carry their own computation information. Classes exhibit large grain size, and translating their contents to rules would require many rules. An obvious advantage over the rule scheme is that elements that conceptually belong together are packaged together into a class, with some control information included. Other frame-based schemes for medical consultation systems include the MDX system (Chandrasekaran *et al.* 1979) and CADUCEUS (Pople 1982).

2.2 Multi-dimensional levels of detail

The term 'level of detail' seems to denote different things to different people. In most schemes, it is used to express problem decomposition only (Nilsson 1971). We present two separate views of abstraction 'level'. These views are related to the fact that all concepts have both IS-A and PART-OF relationships with other concepts. Thus the level of specificity of detail can be controlled by, or examined by traversing, the IS-A hierarchy, while the level of resolution of detail (decomposition in other schemes) is reflected in the PART-OF hierarchy. In Patil *et al.* (1982) only the decomposition view of level is present, while in CADUCEUS (Pople 1982), it seems that the level of specificity is employed and level of resolution is restricted to causal connections. In Wallis and Shortliffe (1982) rule complexity is used, which may be likened to our view of level of resolution; however, its use is restricted to explanation.

2.3 Time and its representation

Several interacting mechanisms are available for the representation of temporal information. This multi-pronged approach differs from other schemes that embody a single type of construct for handling temporal information. The complexity of time necessitates several special mechanisms. Our approach differs from others (Allen 1981; Mittal and Chandrasekaran 1980) in that we were motivated by problems in signal analysis rather than in representing natural language temporal descriptions and their inherent ambiguity and vagueness. It is not clear, for example, what kind of control strategy can be employed along with Allen's scheme of temporal representation. Fagan (1980) is concerned with a temporal interpretation situation. However, there are a number of issues, primarily in control, that are not considered by his system, VM:

- using the rule-based approach, only a data-driven recognition scheme is incorporated, and thus, VM cannot initiate a search for temporally expected events;
 - the handling of noise is not formalized, but is rather ad hoc;
- the complexity of temporal relationships among rules seems limited, and arbitrary groupings of temporal events and their recognition are not addressed;
- expectations in time are table-driven, and no distinction is made between them and default values or expected ranges. Expectations in ALVEN are computed from such information, but current context is taken into account as well so that expectations are tailored for the task at hand;
- partial satisfiability of temporal event groupings cannot be handled.

In addition, Long and Russ (1983) also address the problem of time-dependent reasoning. Their scheme is closer to Fagan's than to ours. The control is data-driven exclusively. Their representation of time, however, shares some similarities with ours in that both points and intervals are used, and special meaning is assigned to the variable 'now'.

A brief description of the representation of time used by ALVEN follows. A TIME_INTERVAL class is defined that contains three slots, namely, start time, end time, and duration. This class can then be included in the structure of any other class and would define its temporal boundaries and uncertainty in those values. Using those slots, the relations before, after, during, etc., (similar to Allen (1981)) are provided. In constraint or default definition, sequences of values (or ranges of values) may be specified using an 'at' operator, so that in effect a piecewise linear approximation to a time-varying function can be included. In this case of course, constraint evaluation must occur at the proper point in time. Tokens of values such as volume or velocity for which use of this operator is appropriate, have two slots, one for the actual value and the other for the time instant at which that value is true. The time instant slot is a dependent slot whose value is set to the value of the special variable 'now' (current time slice). Note that this kind of mechanism could easily be expanded if required to multi-dimensional functions.

Finally, arbitrary groupings of events can be represented. The set construct (which may be used for any type of class grouping, not only for events) specifies elements of a group, names the group as a slot, and has element selection criteria represented as constraints on the slot. Patil *et al.* (1982) described a version of temporal aggregation similar to ours, but do not seem to have a time-line along which selection of values can occur, nor do they distinguish between aggregations of events and sequences of measurements.

Since knowledge classes are organized using the IS-A and PART-OF relations, their temporality is as well. By constructing a PART-OF hierarchy of events, one implicitly changes the temporal resolution of knowledge classes (as long as not only simultaneous events are considered). For example, suppose that the most primitive events occur with durations on the order of seconds. Then groupings of those may define events that occur with durations in the minute range, and then groupings of those again on the order of hours, and so on. Events whose durations are measured using months can be so built up. Yet, many kinds of events cannot be so decomposed, and there is no requirement that all events have such a complete decomposition. Those events, however, are not left hanging, since they will also be related to others in the knowledge base via the IS-A relationship. The control scheme makes use of the temporal resolution with respect to sampling rates and convergence of certainties.

In the following examples, first the TIME_INTERVAL class is shown, followed by the class for the concept of SEQUENCE, followed by a constraint on volume of the left ventricle from the normal left ventricle class, showing the use of the 'at' mechanism for both default and constraint definition, as well as an example of a piecewise linear approximation to the volume vs time function.

```
Example 1
class TIME_INTERVAL with
prerequisites
st: TIME_V such that [st > = 0];
et: TIME_V such that [et > = st];
dependents
dur: TIME_V with dur ← et - st;
end $

Example 2
class SEQUENCE is-a MOTION with
prerequisites
motion_set: set of MOTION such that [
```

```
for all m: (MOTION such that
      [m element-of motion_set])
      verify [
       m.subj = self.subj,
       find m1: MOTION where [
       ml element-of motion set,
       (m1.time_int.st during m.time_int or
       m.time_int.st during m1.time_int)],
       find m2: MOTION where [
         m2 element-of motion_set,
         (m.time_int.st = m2.time_int.et or
         m2.time_int.st = m.time_int.et)]],
       card(motion\_set) > 1,
       strict_order_set(motion_set,time_int.st)];
dependents
  first_mot : MOTION with
    first\_mot \leftarrow earliest\_st(motion\_set);
  last mot: MOTION with
    last\_mot \leftarrow latest\_st(motion set;
  time_int : with time_int ←
    (st of TIME_INTERVAL with st \leftarrow first_mot.time_int.st,
    et of TIME_INTERVAL with et \leftarrow last_mot.time_et);
end $
Example 3
volume: VOLUME_V with
  volume ← (vol of VOLUME_V with
    vol \leftarrow (minaxis.length @ now) ** 3
       default (117 @ m.systole.time_int.st,
           22 @ m.systole.time_int.et,
          83 @ m.diastole.rapid_fill.time_int.et,
         100 @ m.diastole.diastasis.time_int.et,
         117 @ m.diastole.atrial fill.time_int.et)
       such that [
     volume @ m.diastole.time_int.et > = 97
       exception [TOO_LOW_EDV with volume \leftarrow volume ],
     volume @ m.diastole.time_int.et < = 140
       exception [TOO_HIGH_EDV with volume \leftarrow volume ],
     volume @ m.systole.time_int.et > = 20
       exception [TOO_LOW_ESV with volume \leftarrow volume],
     volume @ m.systole.time_int.et < = 27
       exception[TOO_HIGH_ESV with volume \leftarrow volume]],
  time\_inst\ of\ VOLUME\_V\ with\ time\_inst\ \leftarrow now);
```

A few words of explanation are in order. The key words "verify", "find", and "strict_order_set" appear. Their meanings are straightforward: "verify" means match constraints, "find" is the equivalent of "does there exist", and "strict_order_set" is a function that checks to see if a potential motion_set's elements are strictly ordered in time.

2.4 Exceptions and similarity relations

The recording of exceptions to slot filling and constraint matching has proven to be valuable. Exceptions are classes in their own right, with slots to be filled on instantiation, i.e., when raised. Each slot constraint (or group of constraints) of a class may have an associated exception clause. This clause names the type of exception that would be raised on matching failure, and provides a definition for filling the exception's slots, since these slot fillers identify the context within which the exception occurred and play an important role in the determination of the action to take on the exception. Each slot has an implicit exception associated with it for cases where a slot filler cannot be found. Exceptions are used in two ways: (1) to record the matching failures of current hypotheses, recording the failures of the reasoning process; and (2) to assist in directing system attention to other, perhaps more viable hypotheses. The

prototypical exception class is shown below along with one of its specializations, followed by an example from a stroke volume slot. Other examples have already appeared in example 3.

```
Example 4
class EXCEPTION with
dependents
  subj : PHYS OBJ ;
  time_int: TIME_INTERVAL;
  source type: CLASS;
  source_id: INTEGER;
end $
Example 5
class TOO_MUCH_MOTION is-a EXCEPTION with
dependents
  seg: STRING;
  disp: LENGTH_VAL with disp ←
      (len of LENGTH_VAL with
    len ← dist(subj.centroid @ source_id.time_int.st,
         subj.centroid @ source_id.time_int.et),
    time_inst of LENGTH_VAL with time_inst \leftarrow now);
end $
Example 6
stroke vol: VOLUME_V with
  stroke\_vol \leftarrow (vol of VOLUME\_V with
    vol ← self.volume @ m.diastole.time_int.et -
             self.volume @ m.systole.time_int.et
       default(95) such that [
      vol > = 70
         exception [LOW_STROKE_VOLUME with
           volume \leftarrow vol ],
       vol < = 120
         exception [HIGH_STROKE_VOLUME with
           volume \leftarrow vol ]],
    time inst of VOLUME_V with time_inst \leftarrow now);
```

Similarity measures that can be used to assist in the selection of other relevant hypotheses on hypothesis matching failure are useful in the control of growth of the hypothesis space. These measures usually relate classes that together comprise a discriminatory set, i.e., only one of them can be instantiated at any one time. As such, they relate classes that are at the same level of specificity of the IS-A hierarchy, and that have the same IS-A parent classes. Similarity links are components of the frame scheme of Minsky (1975), and a realization of SIMILARITY links as an exception-handling mechanism is presented in Tsotsos et al. (1980) based on a representation of the common and differing portions between two classes. This view is contrasted with the sets of competitors described for the ABEL system (Patil et al. 1982). In that formulation, the level of specificity of the competing set is not represented. Similarity links enable explicit discussion of class comparisons, not only between the connected classes, but also by traversals of several links (Gershon 1982). Thus they are an element of embedded declarative control, and add a different view of class representation, thereby enhancing redundancy of the representation.

The three major components of a SIMILARITY link are (1) the list of target classes (given first), (2) the 'similarities' expression; and (3) the "differences" expression, the time-course of exceptions that would be raised through inter-slot constraints of the source class or in parts of the source class. The similarities represent the important common portions between the source and target classed—during interpretation, the target classes are not active when the SIMILARITY link is being

evaluated; thus, in time-dependent reasoning situations, the components of the target class that are the same as in the source class before activation of the SIMILARITY link, or that the source class may not care about that have already 'passed in time', can be verified using the similarities expression. There is an implicit conjunction of the differences in the exception record, while the similarities form a disjunction. Many SIMILARITY links will be shown in subsequent examples.

2.5 Partial results and levels of description

Partial instances are permitted with an accompanying exception record. More importantly, since instance tokens are produced for each verified hypothesis, and since hypotheses maintain the organization exhibited by the classes that they are formed from, interpretation results also exhibit the same structure. That is, there are levels of description that may be examined as appropriate by a user.

It is important to realize that the instantiation of a hypothesis is achieved only when its certainty has reached a threshold value. (The thresholds are not set in an *ad hoc* fashion, but rather depend on a number of factors relating to the context of interpretation and knowledge structure; see Tsotsos (1984) for details). Thus, even though not all components of a hypothesis have been verified, instantiation may still take place if that hypothesis has significantly more successes than its competitors over the same time period. This would then create a partial instance, including the verified components, the final certainty, and a set of exception records specifying what was not observed.

3.0 The interpretation control structure

3.1 Hypothesize and test: Parallelism and levels of attention

The ALVEN system employs hypothesize and test as the basic recognition paradigm. The activation of a hypothesis sets up an internal goal, that is, that the class from which the hypothesis was formed, try to verify itself. However, activation of hypotheses proceeds along each of five dimensions concurrently, and hypotheses are considered in parallel rather than sequentially. These dimensions are the same class organization axes that are described above. Specifically, we define: goaldirected search to be movement from general to specialized classes along the IS-A dimension, the goal being to find the appropriate sub-class definition for the data in question; model-directed search to be movement from aggregate to component classes along the PART-OF dimension; temporally directed search to be a specific form of model-directed search in that a temporal ordering among components controls the time of activation; failure-directed search to be movement along the SIMILARITY dimension; and data-directed search to be movement from components to aggregates of components upwards along the PART-OF dimension. For a given set of input data, in a single time slice, activation is terminated when none of the activation mechanisms can identify an un-activated viable hypothesis. Termination is guaranteed by virtue of the finite size of the knowledge and the explicit prevention of re-activation of already active hypotheses. The activation of one hypothesis has implications for other hypotheses as well, as will be described below. Because of the multi-dimensional nature of hypothesis activation, the 'focus' of the system also exhibits levels of attention. That is, in its examination, the focus can be stated according to desired level of specificity or resolution (the two are related), discrimination set, or temporal slice.

Each newly activated hypothesis is recorded in a structure

that is similar to the class whose instance it has hypothesized. This structure includes the class slots awaiting fillers, the relationships that the hypothesis has with other hypotheses, and an initial certainty value determined by sharing the certainty with the hypothesis that activates the new hypothesis.

3.2 Goal-directed and model-directed search

Top-down traversal of an IS-A hierarchy, moving downward when concepts are verified implies a constrained form of hypothesize-and-test for more specialized concepts. Similarily, top-down traversal of the PART-OF hierarchy implies a constrained form of hypothesize-and-test for components of classes that reflect greater resolution of detail. These search dimensions are success-driven, as shown in Fig. 2.

Verification of an IS-A parent concept implies that perhaps one of its IS-A siblings applies, while the confirmation of an IS-A sibling implies that its parents must also be true. Multiple IS-A siblings can be activated, but a more efficient scheme would be to activate one of the siblings if all siblings form a mutually exclusive set, or one from several such sets, and then allow failure-directed search to take over. This mechanism will then determine how many siblings in a discriminatory set are viable possibilities. Note that hypotheses are activated for each class in a particular IS-A branch as the hierarchy is being traversed, and thus tokens will be created for each on instantiation. The activation of a hypothesis implies activation of all of its PART-OF components as hypotheses as well. Cycles are avoided since at most one hypothesis for a particular class can exist for each time interval and set of structural components.

In the case of top-down PART-OF hierarchy traversal, the activation of a hypothesis forces activation of hypotheses corresponding to each of its components, i.e., slots. Note that slots may have a temporal ordering, a feature handled by the temporal search mechanism interacting with this one. The search is therefore for all components of a class, increasing the resolution of the class definition.

The MYCIN system (Shortliffe 1976) has only a single search dimension, namely that of depth-first search of the AND/OR tree of rules, while the INTERNIST system (Pople 1977) employs both of these mechanisms in addition to the data-directed search about to be described.

3.3 Data-directed search

The PART-OF hierarchy can also be traversed bottom-up in aggregation mode as shown in Fig. 3. Bottom-up traversal implies a form of hypothesize-and-test, where hypotheses activate other hypotheses that may have them as components, i.e., data-directed search. This form of search is success-driven as well. Activation of hypotheses in this direction implies activation of all IS-A ancestors of new hypotheses as well. Arbitrary hypothesis groupings can be accomplished, but specific groupings can only be recognized if defined as a class.

3.4 Failure-directed search

Failure-directed search is along the SIMILARITY dimension as in Fig. 4, and depends on the exceptions of a particular hypothesis. Typically, several SIMILARITY links will be activated for a given hypothesis, and the resultant set of hypotheses is considered as a discriminatory set, i.e., at most, one of them may be the correct one. Similarity interacts with the PART-OF relationship in that exceptions raised that specify missing slot tokens are handled by the hypothesis' PART-OF parent, the hypothesis that contains the context within which the exception occurred.

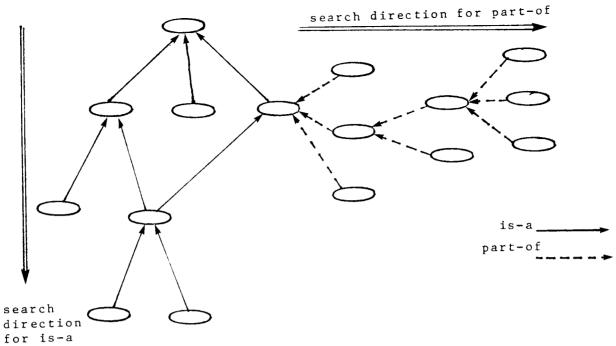


Fig. 2. Goal-directed and model-directed search.

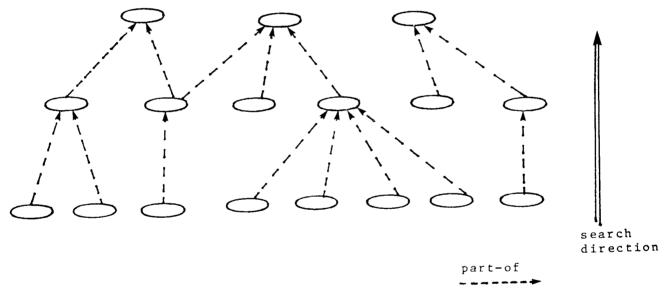


Fig. 3. Data-directed search.

3.5 Temporally directed search

Temporally directed search is automatically activated whenever a class has an IS-A relationship with the SEQUENCE class, and this includes sequences of measurement values. It is, in other words, a special case of model-directed search along the PART-OF dimension (see Fig. 5). (Causal search is a special case of temporal search, since causality implies an existensional dependency as well as a temporal relation. This is present in the CAA system (Shibahara et al. 1983)). Note that elements of a sequence may be compound events, such as other sequences, simultaneous events, or overlapping events. In a sequence, each element of the sequence has a PART-OF relationship with the event class. Thus, on activation of the class, it is meaningless to activate all parts of a sequence at the same time or to expect all measurements of a sequence at the same time. This dimension of search is crucial for the ordering of expectations in time.

3.6 Hypothesis conceptual adjacency

Active hypotheses are related to one another by their 'conceptual adjacencies'. If a knowledge organization relation (IS-A, PART-OF, SIMILARITY, Temporal Precedence) exists between two classes, and hypotheses are active for those two classes such that the hypotheses involve the same set of structural components and time interval (they are attempting to explain the same phenomenon), then the hypotheses also have that same relation. The conceptual adjacency is one of the major components of hypothesis ranking, in that it specifies what kinds of global and local consistencies play a role for a given hypothesis. In fact, the certainty updating scheme only uses information about conceptual adjacency and hypothesis matching.

An interesting result of the use of conceptual adjacencies in hypothesis ranking is that performance of the ranking scheme is

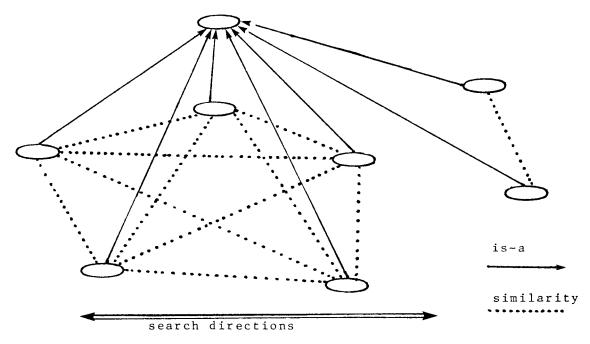


Fig. 4. Failure-directed search.

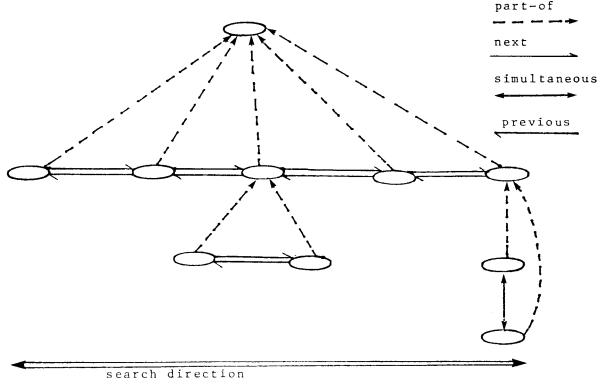


Fig. 5. Temporally directed search.

dramatically improved by their use. In Tsotsos (1984), experiments on the control structure are reported. Those experiments show that for this particular hypothesis certainty updating scheme (and indeed for relaxation labelling schemes in general), the addition of the global constraints to competition exhibited in a discriminatory set of hypotheses, via the IS-A relation, speeds up convergence to correct instantiation.

3.7 Hypothesis matching and hypothesis ranking

The matching result of a hypothesis for the purpose of hypothesis ranking is summarized as either success or failure.

Matching is defined as successful if all slots that should be considered for filling are filled and no matching exceptions are raised. Otherwise, the match is unsuccessful. Using this binary categorization of matching, and the conceptual adjacencies amongst hypotheses, a certainty updating scheme based on relaxation processes (Zucker 1978) is used. Details of this scheme appear in Tsotsos (1984). Basically, hypotheses that are connected by conceptual adjacencies that imply consistency support one another, and those linked by adjacencies that imply inconsistency compete with one another by removing support. The IS-A relationship is in the former group, while the

SIMILARITY relationship is in the latter group. The focus of the system is defined as the set of best hypotheses, at each level of specificity, for each set of structural components being considered in the given time slice. The focus, because of the slow change of certainties inherent in relaxation schemes, exhibits inertia or procrastination, i.e., it does not alter dramatically between certainty updates. Both global and local consistency is enforced through the contributions of hypotheses to one another via their conceptual adjacencies.

3.8 Hypothesis instantiation and deletion thresholds

The use of thresholds is necessitated by the numerical nature of certainties for the instantiation and deletion of hypotheses. These thresholds are not fixed for the lifetime of the system, but rather are dynamic, in that they depend on the number of competing hypotheses in a discriminatory set, and on whether or not the same hypothesis is present in more than one discriminatory set. Sampling rate and noise considerations are also included (see Tsotsos 1984). In particular, the sampling rate depends on the temporal resolution of the discriminatory set, and can thus be variable. Events whose durations are described in months must be samples at that resolution, and those of durations in seconds must be sampled accordingly.

4.0 LV Dynamics knowledge and its representation

Although there is still much work to be done in the determination of the knowledge of LV dynamics, much can be found in current literature that can be incorporated into our formalism. Two examples will be given. This knowledge is used as a starting point for knowledge-base construction only. Moreover, although the exact numerical quantities may differ between imaging techniques, the *qualitative* descriptions do not.

In the series of papers by Gibson and his colleagues (i.e., Gibson et al. (1976); Doran et al. (1978)), several investigations were carried out that determined quantitative aspects of specific LV motions. In the Gibson et al. paper, the segmental motions of the LV during isovolumic relaxation were examined in normal and ischemic LVs using echocardiography in order to determine dynamic differences between these two cases. Without describing technical details of their method, we will briefly summarize their findings. They discovered that in normal LV's an outward wall motion of 1.5-3.0 mm could be present in any region during isovolumic relaxation. In abnormal cases, i.e., patients with coronary artery disease, affected areas show inward motion, 2 mm or more for posterior or apical segments, and any at all for anterior regions, and non-affected areas, because of a compensatory mechanism, may exhibit an increased outward motion of up to 6 mm over normal. The key feature to note here is that the description given does not have a mathematical form at all—it is a combination of quantitative and qualitative measures. The term 'outwards' does not specify any precise direction as long as the motion of the segment is away from the inside of the LV. It is not impossible to set up a mathematical model of this; however, the model will be both cumbersome and will bury the pertinent facts in its equations, so that inspection by a non-sophisticated user becomes impossible. The knowledge class for this information (and more) follows:

```
class N_ISORELAX is-a NO_VOLUME_CHANGE with
prerequisites
  subj : N_LV such that [
    (find ant_mot : NO_TRANSLATION where [
        and mot.subj = self.subj.anterior,
```

```
ant mot.time int = self.time int
  1
find ant mot: OUTWARD where [
  ant mot.subj = self.subj,
  ant mot.time int = self.time_int,
  dist(ant_mot.subj.centroid @ ant_mot.time_int.st,
    ant mot.subj.centroid @ ant mot.time int.et) < 3
  exception [TOO MUCH MOTION with seg ← "anterior",
       direction ← "outward",
    disp ← dist(ant mot.subj.centroid @ ant mot.time_int.st,
       ant mot.subj.centroid @ ant_mot.time_int.et)]
) exception [TOO_MUCH MOTION with seg ← "anterior",
       direction \leftarrow "inward"],
(find post mot: NO TRANSLATION where [
  post_mot.subj = self.subj.posterior,
  post mot.time_int = self.time_int
find post mot: INWARD where [
  post mot.subj = self.subj,
  post_mot.time_int = self.time int,
  dist(post_mot.subj.centroid @ post mot.time int.st,
    post_mot.subj.centroid @ post_mot.time_int.et) < 2
    exception [TOO_MUCH_MOTION with seg ←
     "posterior",
       direction ← "inward",
  disp \leftarrow dist(post\_mot.subj.centroid @ post\_mot.time\_int.st,
    post_mot.subj.centroid @ post_mot.time_int.et)]
find post_mot : OUTWARD where {
  post mot.subj = self.subj,
  post_mot.time_int = self.time_int,
  dist(post_mot.subj.centroid @ post_mot.time_int.st,
     post mot.subj.centroid @ post_mot.time_int.et) < 3
  exception [TOO_MUCH MOTION with seg ← "posterior",
       direction ← "outward",
  dist(post_mot.subj.centroid @ post_mot.time_int.et,
     post mot.subj.centroid @ post_mot.time_int.et)]]
),
 (find ap_mot: NO_TRANSLATION where [
    ap_mot.subj = self.subj.apical,
    ap mot.time int = self.time_int
 find ap_mot : INWARD where [
    ap_mot.subj = self.subj,
    ap mot.time int = self.time_int,
    dist(ap_mot.subj.centroid @ ap_mot.time_int.st,
      ap_mot.subj.centroid @ ap_mot.time_int.et) < 2
      exception [TOO_MUCH_MOTION with seg ← "apical",
        direction ← "inward",
      disp ← dist(ap_mot.subj.centroid @ ap_mot.time_int.st,
        ap_mot.subj.centroid @ ap_mot.time_int.et)]
    1
 or
 find ap_mot : OUTWARD where [
    ap mot.subj = self.subj,
    ap mot.time_int = self.time_int,
    dist(ap mot.subj.centroid@ap_mot.time_int.st,
    ap_mot.subj.centroid@ap_mot.time_int.et) < 3
 exception [TOO_MUCH_MOTION with seg ← "apical"
        direction ← "outward",
    disp \leftarrow dist(ap\_mot.subj.centroid @ ap\_mot.time\_int.st,
        ap mot.subj.centroid @ ap_mot.time_int.et)]]
    ];
```

```
dependents
  time_int : with time_int ← (dur of TIME_INTERVAL with
  dur \leftarrow default(0.093*(30/(0.8*HR))))
    such that [
      time int.st \geq 0.24*(30/(0.8*HR)),
      tim int.et \leq 0.43*(30/(0.8*HR)),
      time_int.dur \geq 0.08*(30/(0.8*HR)),
      time_int.dur \leq 0.12*(30/(0.8*HR))
        exception [TOO LONG_ISORELAX]
    ];
similarity links
  sim_link1: ISCH_AP_ISOVOL_RELAX
    for differences
      d1: TOO_MUCH_MOTION where [
        seg = "apical",
        direction = "inwards",
        time_int = ap_mot.time_int];
      d2: TOO MUCH MOTION where [
        seg = "anterior"
        direction = "outwards",
        disp < 9,
        time_int = ant_mot.time_int];
      d3: TOO_MUCH_MOTION where [
        seg = "posterior",
        direction = "outwards",
        disp < 9,
        time_int = post_mot.time_int];;
  sim_link2: ISCH_ANT_ISOVOL_RELAX
    for differences:
      d1: TOO MUCH MOTION where [
        seg = "anterior",
        direction = "inwards",
        time_int = ant_mot.time_int];
      d2: TOO_MUCH_MOTION where [
        seg = "apical",
        direction = "outwards",
        disp < 9,
        time_int = ap_mot.time_int];
      d3: TOO_MUCH_MOTION where [
        seg = "posterior",
        direction = "outwards",
      disp < 9,
      time int = post_mot.time_int];;
sim_link3: ISCH_POST_ISOVOL_RELAX
  for differences:
    d1: TOO_MUCH_MOTION where [
      seg = "posterior",
      direction = "inwards".
      time_int = post_mot.time_int];
    d2: TOO_MUCH_MOTION where [
      seg = "anterior"
      direction = "outwards",
      disp < 9,
      time_int = ant_mot.time_int];
    d3: TOO MUCH MOTION where [
      seg = "apical",
      direction = "outwards",
      disp < 9,
      time_int = ap_mot.time_int];;
end
```

The definition states that for a normal isovolumic relaxation phase to be recognized, normal motions for each segment must be present. There are three main clauses in the definition. The first defines the expected normal motion of the anterior segment, the second for the posterior segment, and the third for the remaining segment, the apical one. So for example, in the first clause, the definition reflects Gibson's characterization: the

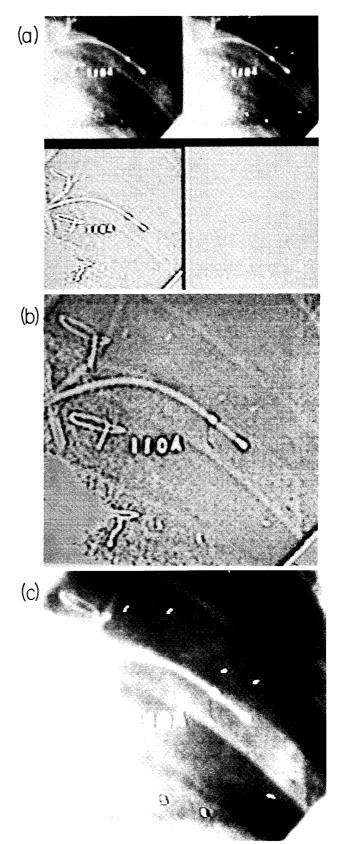


Fig. 6. A typical image and 'blind' marker finding.

anterior segment during this phase must either not display any translational movement, or could display an outward motion of displacement of less than 3 mm. A larger displacement than this in the outwards direction would be recorded as the exception TOO_MUCH_MOTION, with specific additional contextual

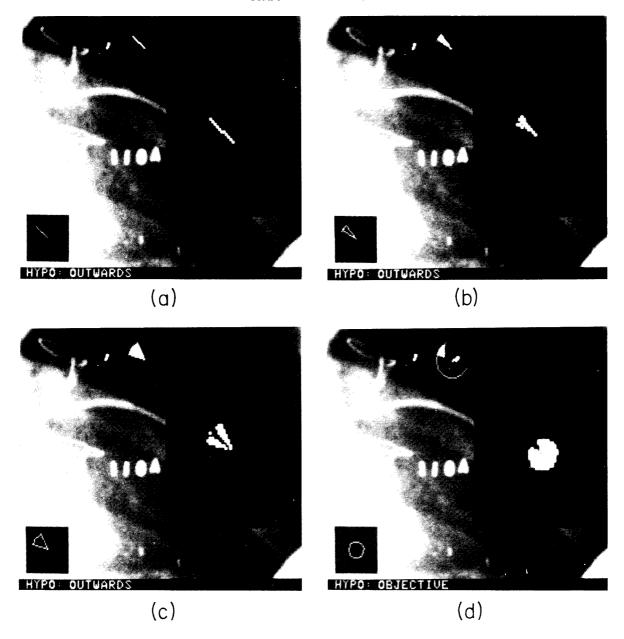


Fig. 7. Guidance for marker finding from hypotheses.

information recorded as well. In the matching of class definitions to actual observed motions, matching failures are recorded as exceptions. If the anterior segment was displaying motion and was not outwards, then it must be inwards, and this fact too would be recorded as an exception. The dependent portion specifies relevant timing information for the temporal placement of the phase within the left ventricular cycle. HR is in units of beats per second, so that the right-hand side of the timing expressions is in units of number of images. Also, using the information derived from Gibson et al. (1976), the similarity links provide definitions of the constraints that must be found if a possible ischemic segment is to be recognized. Note that only the connections to possible ischemic states detectable by considering only the characteristics of the isovolumic relaxation phase are included above; a set of similarly formed constraints would have to be present for other disease states as well, for those cases where the isovolumic relaxation phase plays a role in their definition. 'sim link2' relates the normal phase to the motion of an abnormal apical segment exhibiting the effects of ischemia. This, according to Gibson's definition, is shown by either the apical region itself having too much inward motion during this phase, and (or) one of the other regions (posterior or anterior) exhibiting too much outward motion during the phase. Note that the set of differences does not define a necessary set; any one of the conditions is sufficient.

If should be clear that the above is not complete; it requires the remainder of the definitions for the other phases and motions, since the entire definition of each class of LV motion is defined as a hierarchy of abstraction, each level adding more detail to the previous one. Some of the types of information that are represented are: volume changes where known for normal phases, ejection fractions, for example; measures of degrees of abnormalities, derived heuristically; and others.

A second body of knowledge of the form necessary for interpretation can be found in Fujii *et al.* (1979). These researchers investigated eight different clinical cardiac disease

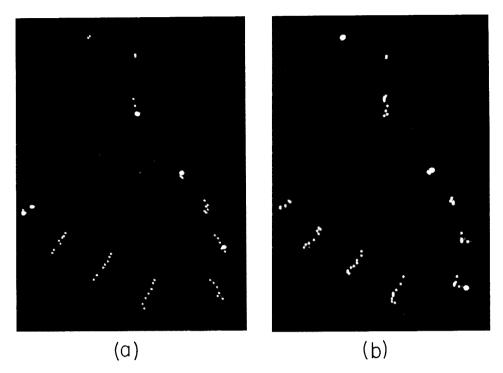


Fig. 8. Inward and outward motions of an LV cycle.

states with the intent of discovering posterior-wall motion differences and similarities among the diseases, as well as global LV characteristics. The diseases were: pericarditis, congestive cardiomyopathy, hypertrophic cardiomyopathy, valvular aortic stenosis, aortic insufficiency, mitral stenosis, mitral insufficiency, and systemic hypertension. Normal LV's were also studied. The measurements made for each of the above LV states were: stroke volume, rapid filling volume, slow filling volume, atrial filling volume, the percent filling for each of the previous three phases with respect to the stroke volume, posterior wall excursion in total, and for each of the three phases of diastole, as well as the percentage excursion in each phase diastolic posterior wall velocity, rapid filling rate, LV end diastolic dimension, and ejection fraction. It is, of course, difficult to verify their results. However, they are important for they provide at least a starting point for the further elaboration and verification of such detailed dynamic information. In addition to the large amount of numerical information that they derived, they attached to the significant findings qualitative descriptors, such as whether or not this quantity should be higher or lower than in the normal case. This was rather fortunate from our point of view: the representational formalism that we had designed can handle description via common components and differences very well, and uses such information to advantage during the decision phases of the interpretation. It should be clear from the previous example how such information would be included into the representation, and this fact alone raises another important advantage of this scheme. The addition of information into a mathematical model may require a complete re-definition of the model. In our case, information is easily inserted, as long as one understands the semantics of the representation.

5.0 An example

The initial evaluation is done on a cine contrast representation; each patient has a permanent volume correction factor for both diastole and systole that accounts for the shell of muscle enclosed by the contour created by connecting the markers (Alderman et al. 1976). Interpolation is used for variations in this correction over time. Nine markers on the LV wall, and two on the aortic valve edges constitute the LV outline from which volume calculations are done, using an area-length formula that was devised for this purpose. Figure 6 displays an actual image with the stages of image analysis that lead to 'blind' marker finding, that is, without any sort of guidance as to expected marker location. The first stage involves filtering the image with a Marr—Hildreth like operator (Hildreth 1980). Zero-crossings with their standard definition, however, do not lead to useful image tokens because of the nature of the X-ray images and their low contrast. A specially tuned version of the Marr-Hildreth operator was then used to extract the markers. This operator was tuned such that the size and shape of the marker was reflected in the center of the operator with the surround enveloping this center. The results of this are then superimposed on the original image, in order to highlight the markers. These two steps are expanded in Fig. 6b and c.

Guidance, however, is an integral feature of the framework, namely, during the hypothesization of motion classes, the hypotheses themselves can be used to predict expected motion characteristics for the markers, segment, and entire left ventricle. Figure 7 then shows the kind of predictions that an 'outwards' motion hypothesis creates and the guidance it provides. Note that for this example 'outwards' refers to outwards motion of the marker with respect to the segment, not to the ventricle. Clearly, for this case the marker is not found on that path. The hypothesis structure is then modified to enclose a larger space, corresponding to a relaxation of the constraints of the hypotheses, until it is found. The same marker-finding process described earlier is used, but only in the prediction window. Four images are shown corresponding to the four predictions generated until this marker is found. In addition to the examination of a very small image subset for each marker,

```
PERIMETER CHANGE—time interval (0, 8)
marker 5 exhibits:
TRANSLATING—time interval (0, 5)
                                                                                     rate (mm/s) \rightarrow 45, -60, 30, -45, -45, -30, -30, 90
                                                                                     specializations:
rate (mm/s) \rightarrow 60, 21, 33, 45, 51
trajectory (radians) \rightarrow 4.71, 2.36, 0.46, 1.24, 2.18
                                                                                       LENGTHENING during (0, 1)
                                                                                       SHORTENING during (1, 2)
specializations:
  OUTWARDS wrt ANTERIOR during (0, 1)
                                                                                       LENGTHENING during (2, 3)
  INWARDS wrt ANTERIOR during (1, 2)
                                                                                       SHORTENING during (3, 7)
  OUTWARDS wrt ANTERIOR during (2, 3)
                                                                                       LENGTHENING during (7, 8)
                                                                                     PERIMETER CHANGE—time interval (9, 10)
TRANSLATING—time interval (6, 10)
                                                                                     rate (mm/s) \rightarrow -30
rate (mm/s) \rightarrow 15, 33, 42, 15
trajectory (radians) \rightarrow 1.24, 4.19, 5.50, 1.24
                                                                                     specializations:
                                                                                       SHORTENING during (9, 10)
specializations:
  INWARDS wrt ANTERIOR during (6, 7)
                                                                                     PERIMETER CHANGE—time interval (13, 16)
  OUTWARDS wrt ANTERIOR during (7, 9)
                                                                                     rate (mm/s) \rightarrow 30, -30, 45, 45
                                                                                     specializations:
TRANSLATING—time interval (14, 15)
rate (mm/s) \rightarrow 15
                                                                                       LENGTHENING during (13, 14)
trajectory (radians) → 1.24
                                                                                       SHORTENING during (14, 15)
                                                                                       LENGTHENING during (15, 16)
specializations:
  INWARDS wrt ANTERIOR during (14, 15)
                                                                                     others
                                                                                     NO TRANSLATION during (1, 3)
NO MOTION during (5, 6)
                                                                                     NO TRANSLATION during (8, 9)
                                                                                     NO PERIMETER CHANGE during (8, 9)
NO MOTION during (10, 14)
                                                                                     NO PERIMETER CHANGE during (10, 13)
NO MOTION during (15, 16)
                                                                                     NO TRANSLATION during (11, 13)
exceptions to normal detected:
MODERATELY HYPOKINETIC—CONTRACTION
                                                                                     exceptions to normal detected:
  wrt ANTERIOR during (1, 2)
                                                                                     SEVERELY HYPOKINETIC—CONTRACTION
                                                                                       wrt VENTRICLE during (2, 6)
ANTERIOR segment exhibits:
                                                                                     TOO SHORT SYSTOLE during (7, 7)
TRANSLATING—time interval (0, 1)
                                                                                     MILDLY POOR SYSTOLE during (7, 7)
rate (mm/s) \rightarrow 45
                                                                                     SEVERELY HYPOKINETIC—EXPANSION
trajectory (radians) \rightarrow 4.71
                                                                                       wrt VENTRICLE during (8, 14)
specializations:
  INWARDS wrt VENTRICLE during (0, 1)
                                                                                     APICAL segment exhibits:
                                                                                     TRANSLATING—time interval (1, 6)
TRANSLATING—time interval (3, 8)
                                                                                     rate (mm/s) \rightarrow 33, 33, 60, 48, 33
rate (mm/s) \rightarrow 30, 15, 21, 15, 15
                                                                                     trajectory (radians) \rightarrow 2.08, 1.05, 1.24, 1.99, 2.08
trajectory (radians) \rightarrow 1.24, 1.24, 2.36, 2.36, 4.71
                                                                                     specializations:
specializations:
                                                                                       INWARDS wrt VENTRICLE during (1, 6)
  INWARDS wrt VENTRICLE during (3, 8)
TRANSLATING—time interval (9, 11)
rate (mm/s) \rightarrow 15, 15
                                                                                     TRANSLATING-time interval (7, 10)
trajectory (radians) \rightarrow 1.24, 1.24,
                                                                                     rate (mm/s) \to 60, 51, 15
specializations:
                                                                                     trajectory (radians) \rightarrow 4.71, 4.09, 3.14
  OUTWARDS wrt VENTRICLE during (9, 11)
                                                                                     specializations:
                                                                                       OUTWARDS wrt VENTRICLE during (7, 9)
TRANSLATING—time interval (13, 16)
                                                                                       INWARDS wrt VENTRICLE during (9, 10)
rate (mm/s) \rightarrow 15, 15, 15, 15
trajectory (radians) \rightarrow 0.00, 3.14, 0.00, 0.00
                                                                                     TRANSLATING—time interval (11, 14)
specializations:
                                                                                     rate (mm/s) \rightarrow 33, 15, 21
  OUTWARDS wrt VENTRICLE during (14, 16)
                                                                                    trajectory (radians) \rightarrow 5.81, 4.71, 5.50
                                                                                     specializations:
VOLUME CHANGE—time interval (0, 16)
                                                                                       OUTWARDS wrt VENTRICLE during (11, 14)
rate (ml/s) \rightarrow -1.2, -66, 33, -48, -30, -12, -3, 27, 12, -12, 21, 33, -6, -6, 2.1, 39, 39
                                                                                     TRANSLATING—time interval (15, 16)
specializations:
                                                                                     rate (mm/s) \rightarrow 33, 33
  CONTRACTING during (0, 1)
                                                                                     trajectory (radians) \rightarrow 5.81, 5.81
  UNIFORMLY CONTRACTING during (0, 2)
                                                                                    specializations:
  SYSTOLE during (3, 6)
                                                                                       OUTWARDS wrt VENTRICLE during (15, 16)
  EXPANDING during (2, 3)
  UNIFORMLY CONTRACTING during (3, 5)
                                                                                     VOLUME CHANGE—time interval (0, 6)
                                                                                    rate (ml/s) \rightarrow -12, -72, -24, -60, -42, -36
  CONTRACTING during (6, 7)
  DIASTOLE during (7, 9)
                                                                                     specializations:
                                                                                       CONTRACTING during (0, 1)
  CONTRACTING during (9, 10)
                                                                                       UNIFORMLY CONTRACTING during (0, 2)
  DIASTOLE during (10, 12)
  CONTRACTING during (12, 14)
                                                                                       SYSTOLE during (1, 6)
  DIASTOLE during (14, 16)
                                                                                       UNIFORMLY CONTRACTING during (3, 6)
```

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```
VOLUME CHANGE—time interval (7, 16)
                                                                   PERIMETER CHANGE—time interval (13, 16)
 rate (ml/s) \rightarrow 54, 24, 15, 15, 48, 36, 9, -15, 45, 45
                                                                  rate (mm/s) \rightarrow 15, 60, 15, 15
 specializations:
                                                                  specializations:
   DIASTOLE during (7, 14)
                                                                     LENGTHENING during (13, 16)
   UNIFORMLY EXPANDING during (7, 8)
   UNIFORMLY EXPANDING during (9, 13)
                                                                  NO PERIMETER CHANGE during (2, 3)
   CONTRACTING during (14, 15)
                                                                  NO MOTION during (6, 7)
   DIASTOLE during (15, 16)
                                                                  NO PERIMETER CHANGE during (12, 13)
 PERIMETER CHANGE—time interval (0, 6)
                                                                  LEFT VENTRICLE exhibits:
 rate (mm/s) \rightarrow 15, -75, -30, -60, -45, -45
                                                                  TRANSLATING—time interval (0, 6)
 specializations:
                                                                  rate (mm/s) \rightarrow 15, 33, 15, 33, 1, 21
   LENGTHENING during (0, 1)
                                                                  trajectory (radians) \rightarrow 4.71, 1.05, 1.24, 1.05, 1.24, 2.36
   SHORTENING during (1, 6)
                                                                  TRANSLATING—time interval (7, 15)
 PERIMETER CHANGE—time interval (7, 16)
                                                                  rate (mm/s) \rightarrow 15, 15, 15, 15, 15, 15, 15
 rate (mm/s) \rightarrow 15, 30, 30, 30, 60, 45, 15, 15, 15, 15
                                                                  trajectory (radians) \rightarrow 4.71, 4.71, 4.71, 3.14, 4.71, 3.14,
 specializations:
                                                                     0.00, 4.71
   LENGTHENING during (7, 13)
   SHORTENING during (13, 14)
                                                                  VOLUME CHANGE—time interval (0, 16)
   LENGTHENING during (14, 16)
                                                                  rate (ml/s) \rightarrow -57, -216, -75, -168, -186, -138, 2, 120, 57, 54,
                                                                     120, 162, 90, 27, 45, 90, 90
                                                                  specializations:
 NO TRANSLATION during (0, 1)
                                                                     UNIFORMLY CONTRACTING during (0, 1)
 NO MOTION during (6, 7)
NO TRANSLATION during (10, 11)
                                                                     SYSTOLE during (1, 6)
                                                                     UNIFORMLY CONTRACTING during (2, 6)
 NO TRANSLATION during (14, 15)
                                                                     UNIFORMLY EXPANDING during (7, 11)
 exceptions to normal detected:
                                                                     DIASTOLE during (7, 16)
 SEVERELY HYPOKINETIC—EXPANSION
   wrt VENTRICLE during (10, 11)
                                                                     UNIFORMLY EXPANDING during (12, 14)
                                                                     UNIFORMLY EXPANDING during (15, 16)
 SEVERELY HYPOKINETIC—EXPANSION
   wrt VENTRICLE during (14, 15)
                                                                  PERIMETER CHANGE—time interval (0, 6)
                                                                  rate (mm/s) \rightarrow 15, -150, 15, -165, -165, -105
 POSTERIOR segment exhibits:
                                                                  specializations:
TRANSLATING—time interval (0, 6)
                                                                    LENGTHENING during (0, 1)
rate (mm/s) \rightarrow 15, 48, 33, 21, 33, 33
                                                                    SHORTENING during (1, 2)
trajectory (radians) \rightarrow 1.24, 0.95, 1.05, 0.77, 1.05, 1.05
                                                                    LENGTHENING during (2, 3)
specializations:
                                                                    SHORTENING during (3, 6)
  INWARDS wrt VENTRICLE during (0, 3)
   OUTWARDS wrt VENTRICLE during (3.4)
                                                                  PERIMETER CHANGE—time interval (7, 8)
  INWARDS wrt VENTRICLE during (4, 6)
                                                                  rate (mm/s) \rightarrow 90
                                                                  specializations:
TRANSLATING—time interval (7, 16)
                                                                    LENGTHENING during (7, 8)
rate (mm/s) \rightarrow 15, 30, 21, 48, 21, 21, 15, 15, 15, 15
trajectory (radians) \rightarrow 4.71, 4.71, 3.92, 3.92, 3.92,
                                                                  PERIMETER CHANGE—time interval (9, 16)
  3.92, 4.71, 0.00, 3.14, 3.14
                                                                  rate (mm/s) \rightarrow 30, 75, 150, 60, 15, 60, 60, 60
specializations:
                                                                  specializations:
  OUTWARDS wrt VENTRICLE during (8, 14)
                                                                    LENGTHENING during (9, 16)
  INWARDS wrt VENTRICLE during (14, 15)
                                                                  WIDTH CHANGE—time interval (0, 16)
VOLUME CHANGE—time interval (0, 6)
                                                                  rate (mm/s) \rightarrow -15, -15, -60, -15, -60, -60, -60, -60, -60, 60, 75.
rate (ml/s) \rightarrow -33, -90, -15, -75, -96, -78
                                                                    45, 45, 45, -15, -15
specializations:
  SYSTOLE during (1, 6)
                                                                  LENGTH CHANGE—time interval (0, 16)
                                                                  rate (mm/s) \rightarrow 30, -45, -15, -60, -60, -30, -30, 45, 15, 15, 15,
VOLUME CHANGE—time interval (7, 16)
                                                                    45, 45, 45, 45, 45, 45
rate (ml/s) \rightarrow 5, 6, 27, 75, 111, 21, 15, 60, 21, 21
specializations:
                                                                  others
  DIASTOLE during (7, 16)
                                                                  ISOMETRIC CONTRACTION during (0, 1)
  UNIFORMLY EXPANDING during (9, 12)
                                                                  NO TRANSLATION during (6, 7)
                                                                  NO PERIMETER CHANGE during (6, 7)
PERIMETER CHANGE—time interval (0, 2)
                                                                  NO PERIMETER CHANGE during (8, 9)
rate (mm/s) \rightarrow -45, -15
specializations:
                                                                  NO TRANSLATION during (15, 16)
  SHORTENING during (0, 2)
                                                                  exceptions to normal detected:
                                                                  MILDLY DYSKINETIC—CONTRACTION during (3, 4)
PERIMETER CHANGE—time interval (3, 6)
                                                                  ISCHEMIC ANTERIOR ISOMETRIC RELAXATION during (6, 7)
rate (mm/s) \rightarrow -30, -90, -15
                                                                  SEVERELY POOR SYSTOLE during (7, 7)
specializations:
                                                                  MODERATELY DYSKINETIC—EXPANSION during (9, 15)
  SHORTENING during (3, 6)
PERIMETER CHANGE—time interval (7, 12)
rate (mm/s) \rightarrow -45, -30, 30, 75, 75
specializations:
  SHORTENING during (7, 9)
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LENGTHENING during (9, 12)

this process of prediction-verification also provides important feedback for other levels of the system. This marker-finding process is guaranteed to always find a marker because the default process is the 'blind' one referred to above. Figure 8 shows the sequence of marker motions for a complete cycle (a different cycle than the one from which the preceding images were taken.)

ALVEN is capable of reporting on LV performance at marker, segment, and global LV levels of detail. Relative directions, motion extents, rates of change, and temporal relationships are described both numerically and symbolically. Anomalies are detected by using the appropriate heuristic or by comparisons to accepted normal performance. Anomalies such as asynchrony, hypokinesis, dyskinesis, too slow or too fast rate of change of volume with respect to the LV phase, too long or too short phase duration, or degree of anomaly are considered.

An example of marker motions is shown in Fig. 8 a and b, for a patient from the Cardiovascular Unit at Toronto General Hospital. Figure 8a shows the contraction phase, while Fig. 8b shows the expansion phase. This particular example was assessed by the radiologists with respect to motion anomalies: the radiologist reported that the anterior segment was hypokinetic, and the remaining segments exhibited normal motion. A portion of the output of the ALVEN system for this particular film (taken at 30 images per second, 17 images in all) is shown in Fig. 9. Let us highlight some of the important points of this analysis. Firstly, a short summary of how to read the example is necessary. For each physical entity that the system knows about, which is in this case the markers, the segments, and the LV as a whole, a short summary of the motions observed is produced. This has been abbreviated because of space limitations in the following way: descriptions for the aortic clips were deleted, as were the descriptions for all of the markers save for marker 5. The remaining motions would have a form similar to those for the other markers. Each motion has a descriptive term, a possible referent where necessary (for example, 'INWARD' motion is not semantically complete without saying inwards with respect to some other object that has an inside, usually defined by the geometric centroid), quantitative values where appropriate (clearly a calibration phase is necessary), and a time interval or instant at which it was recognized. Time is noted in image units. The range of descriptive terms that ALVEN can understand is apparent from the example. Descriptions are shown for only one marker (5), and for each segment, and for the left ventricle. The motion of marker 5 is of particular interest for this example.

Secondly, the example of the knowledge for isovolumic relaxation given earlier is relevant here. The motions exhibited by the anterior segment (that is, there is a small inward motion during that phase, as shown by the description at time interval (6, 7), because of an inward motion of marker 5 during that interval, and further evidenced by the volume contraction noted in the segment description), cause that chunk of knowledge to be activated and verified. The result is the descriptive term 'ISCHEMIC ANTERIOR ISOVOLUMIC RELAXATION'. which can be found in the description of the motions of the left ventricle. In addition, it will usually be true that if one segment is not performing up to par (notice the number of HYPO-KINESIS instances detected—the great majority are present for the anterior segment thus confirming the radiologist's report), then the overall performance of the ventricle must be impaired as well. This can be seen by the instances of 'POOR SYSTOLE' that appear. These are confirmed independently using volume change information.

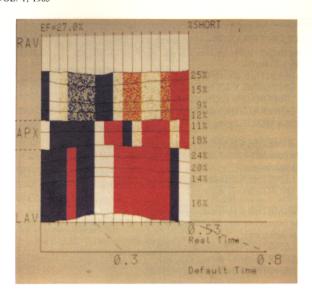


Fig. 10. ALVEN'S graphic display of the evaluation in Fig. 9.

Some other interesting descriptive terms are briefly described. UNIFORM CONTRACT/EXPAND—for this to be detected, the object considered must have a decreasing volume, and all of its markers/segments (depending on the level of description) must be moving in the proper direction. So for a uniform contraction at the LV level, the three segments must all be moving inwards and the overall volume of the LV must be decreasing. HYPOKINESIS—can only be noted if all markers/ segments are moving in the same direction, and a comparison of their relative motions reveals one that is lagging behind. Note that the use of the term hypokinesis does not make sense if all markers are not moving in the same direction, since this is a term describing anomalies of motion extent. If they are all moving too slowly, then no anomaly is detected at the marker level, but it is detected at the segment level. If in turn, all segments are exhibiting small motion extents, no hypokinesis is noted at all, however, serious performance problems will be noted, because the volume changes will be lower than normal. The detection of hypokinesis is purely relational. Note, however, that it is not necessarily so. The data in Fujii et al. (1979) do provide some quantitative information on normal and abnormal extents for the posterior segment; these will be incorporated into the representation. However, the relational approach is a valid one when one is lacking information.

No constraints are currently in place for length changes, i.e., normal or abnormal circumferential shortening, although examples are shown of how such changes are detected.

It should be apparent that the amount of information reported is large, and that this is not a desirable characteristic for a medical consultation system. Therefore, a simple, graphic display has been devised that captures much of the important information required for appropriate analysis. This display is presented in Fig. 10. A brief explanation is in order. Imagine that the ventricle is opened up along the circumference and laid flat along the vertical axis with the right side of the aorta on the bottom, the apex in the middle, and the left side of the aorta at the top. Time is the horizontal axis. For each time interval (image pair of the film), and for each segment, a summary is displayed in terms of whether or not the segment was moving inwards (blue), outwards (red), or was not moving (white). Remember that these are motions relative to the ventricular centroid. The yellow dotting represents hypokinesis, with the more densely concentrated dots representing increasing levels

of severity. The black lines traversing the plot horizontally are the marker paths in time, useful for viewing circumferential shortening effects. Finally, percent shortening for each marker with respect to the ventricular centroid are provided on the right side, along with ejection fraction. This display is particularly clear in revealing temporal relationships of a variety of types.

If these evaluations are compared with those of the radiologist, it can be seen that there is infinitely more detail present in ALVEN's evaluation, yet it is completely consistent with the radiologist's opinion. This has also been borne out in several other examples. Moreover, this analysis is repeatable and objective. Although there is much knowledge refinement required before ALVEN's knowledge base is as competent in general as a good radiologist/cardiologist, the value of the enhanced evaluation is clear.

6.0 Discussion

The knowledge organization dimensions of generalization/ specialization (IS-A), aggregation/decomposition (PART-OF), mutual exclusion (SIMILARITY), and Temporal Precedence have all appeared previously in the representational literature. with the first two receiving the lion's share of attention (see Brachman 1979, 1982; Levesque and Mylopoulos 1979). The arguments for their use have been mostly qualitative, that is, these dimensions seem to have desirable formal properties and lend themselves naturally to the construction of large knowledge bases. In this work, we have shown that not only are these aspects important, but also that each representational dimension has a distinct role to play in an interpretation scheme. In fact, they each have two important roles. One role is that of enabling multiple, interacting search mechanisms. This function should not be underestimated. Rule-based recognition paradigms, for example, only offer a single dimension of search. As pointed out in Aiello (1983), such systems suffer from serious problems owing to the one-dimensionality of the inference procedure. The conclusion that we draw from this report is that goaldirected, data-directed, and model-directed inference mechanisms most effectively can compensate for the deficiencies of each other if used in concert. For example, a data-directed scheme considers all the data and tries to follow through on every event generated. It can be non-convergent, can only produce conclusions that are derivable directly or indirectly from the input data, and cannot focus or direct the search toward a desired solution. The goal-directed strategy is easy to understand and implement, and at each step of the execution the next step is pre-determined. Rules are evaluated in the same order regardless of input data. It is thus inefficient and cannot exhibit a focus with respect to the problem being solved since there is no mechanism that determines what is important and what is not. Finally, the model-directed approach, although the most efficient and the one that exhibits correct foci of problemsolving activity, has the disadvantage that its conclusions depend heavily on the availability of the correct model and initial focus. An incorrect initial focus will lead it to the examination of useless and incorrect analyses and will cause some perhaps relevant data to be ignored.

In our scheme, several dimensions of inference are included, each driven by the semantics of one of the organizational dimensions. They are integrated with one another so that each dimension of search compensates for the failings of another, and thus as a whole offers a rich and robust framework.

Further, each organization dimension offers distinct and necessary contributions to the updating of hypothesis certainty within a relaxation framework, and to the maintenance of consistency within an interpretation. Not only have the knowledge organization dimensions been integrated within a relaxation labelling process, driving the definitions of neighbourhood, compatibilities, weights, and consistency in an intuitive yet concrete fashion, but also the following results on the relaxation process have emerged:

- IS-A, besides offering a definition of global consistency of hypothesis' certainty, plays the role of speeding up the convergence of results. This is an important role, since it allows smaller temporal sampling rates. Because of inheritance, the problem posed by the propagation of results through the network disappears. IS-A also has an important part in the graceful recovery from poor predictions. Finally, feedback imposed by the IS-A hierarchy increases the stability of the cooperative process and partly compensates for the effects of noise disturbances and parameter variations, important considerations for the non-linear relaxation scheme.
- SIMILARITY plays the discrimination role, and is the only mechanism that allows for competition between hypotheses, enabling 'best choice' selection. In conjunction with the exceptions that drive SIMILARITY activations, this is a strong feedback mechanism, enhancing the stability of the cooperative process. Moreover, it is central to the definition of temporal sampling rate and of compatibility values.
- PART-OF is the mechanism that permits the selection of the stronger of two equally consistent hypotheses based on the strength of their components.
- Temporal Precedence assists in the discrimination of proper temporal order, important for temporal predictions, and temporal 'gluing' of events into higher order ones.

Such strong evidence for the use of knowledge organization axes, besides for knowledge access, during interpretation has been lacking in past works, and supports our claim that knowledge organization has far more to offer than has previously been evident.

7.0 Conclusions

A methodology has been presented for the knowledge-based interpretation of continuous time-varying signals. The key idea is that knowledge organization dimensions play several major roles beyond their access and structuring properties. We have shown that significant advantages result in both sophistication of the reasoning process for spatio-temporal data as well as in the formalization of a certainty-updating process rooted in relaxation processes. An example of the practical implementation of this methodology was shown with the examples of ALVEN's knowledge base and analysis. ALVEN is currently implemented on a VAX 11/780 running Berkeley UNIX 4.2 in the C language. The example shown above requires about 30 min. of CPU time, including the generation of the displays.

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