York University
CSE 2001 – Unit 6.0 Proof Systems and NP
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Don't cheat by looking at these answers prematurely.

1. What did Godel proof about proof systems?
   Give few sentence intuition of the proof.

   - Answer: Given any proof system there is a statement that is either not proved or is proved incorrectly.
     Godel's proof: If there was a proof system S, then the sentence Φ = "Φ does not have a proof in S" is a problem. If Φ is true, then what its says is that it does not have a proof. If it is false, then what its says is that it does have proof.
     Turing's proof: Such a proof system would lead to an algorithm that would decide whether a math sentence is true or not which would lead to an algorithm that decides the halting problem.

2. NP

   (a) How do you convince your boss that the computational problem he wants you to solve is likely too hard to solve in general?

   - Answer: Show that is is NP-complete (or even worse undecidable).

   (b) What does it mean for a problem P to be in NP (Non-Deterministic Polynomial Time)?

   - Answer: Given an instance and a solution, it is easy to test its validity.
     $\exists$ poly time verifier $Valid$ such that
     $\forall I$, [$I \in P$ iff $\exists S$ $Valid(I, S)$].

   (c) Why is 3-Col in NP?

   - Answer: Because given a graph G as input and a colouring as a solution S from your fairy god mother, it is easy to check that S is a valid colouring of your graph G.

   (d) What does it mean for a problem P to be NP-Complete.

   - Answer: It is in NP and
     it is as hard as any problem in NP and is in NP,
     i.e. $\forall P' \in NP$ $P' \leq_{poly} P$.